



## DEPARTMENT OF MECHANICAL AND ENGINEERING

### *COURSE MATERIALS*



### *PH100 ENGINEERING PHYSICS*

#### VISION OF THE INSTITUTION

To mould true citizens who are millennium leaders and catalysts of change through excellence in education.

#### MISSION OF THE INSTITUTION

NCERC is committed to transform itself into a center of excellence in Learning and Research in Engineering and Frontier Technology and to impart quality education to mould technically competent citizens with moral integrity, social commitment and ethical values.

We intend to facilitate our students to assimilate the latest technological know-how and to imbibe discipline, culture and spiritually, and to mould them in to technological giants, dedicated research scientists and intellectual leaders of the country who can spread the beams of light and happiness among the poor and the underprivileged.

#### ABOUT DEPARTMENT

- ◆ Established in: 2002
- ◆ Course offered : B.Tech in Mechanical Engineering
- ◆ Approved by AICTE New Delhi and Accredited by NAAC
- ◆ Affiliated to the University of Dr. A P J Abdul Kalam Technological University.

## **DEPARTMENT VISION**

Producing internationally competitive Mechanical Engineers with social responsibility & sustainable employability through viable strategies as well as competent exposure oriented quality education.

## **DEPARTMENT MISSION**

1. Imparting high impact education by providing conducive teaching learning environment.
2. Fostering effective modes of continuous learning process with moral & ethical values.
3. Enhancing leadership qualities with social commitment, professional attitude, unity, team spirit & communication skill.
4. Introducing the present scenario in research & development through collaborative efforts blended with industry & institution.

## **PROGRAMME EDUCATIONAL OBJECTIVES**

**PEO1:** Graduates shall have strong practical & technical exposures in the field of Mechanical Engineering & will contribute to the society through innovation & enterprise.

**PEO2:** Graduates will have the demonstrated ability to analyze, formulate & solve design engineering / thermal engineering / materials & manufacturing / design issues & real life problems.

**PEO3:** Graduates will be capable of pursuing Mechanical Engineering profession with good communication skills, leadership qualities, team spirit & communication skills.

**PEO4:** Graduates will sustain an appetite for continuous learning by pursuing higher education & research in the allied areas of technology.

## **PROGRAM OUTCOMES (POS)**

**Engineering Graduates will be able to:**

1. **Engineering knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
  2. **Problem analysis:** Identify, formulate, review research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
  3. **Design/development of solutions:** Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
  4. **Conduct investigations of complex problems:** Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
  5. **Modern tool usage:** Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
  6. **The engineer and society:** Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
  7. **Environment and sustainability:** Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
  8. **Ethics:** Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
  9. **Individual and teamwork:** Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
  10. **Communication:** Communicate effectively on complex engineering activities with the engineering
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community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

11. **Project management and finance:** Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
12. **Life-long learning:** Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

### **PROGRAM SPECIFIC OUTCOMES (PSO)**

**PSO1:** graduates able to apply principles of engineering, basic sciences & analytics including multi variant calculus & higher order partial differential equations..

**PSO2:** Graduates able to perform modeling, analyzing, designing & simulating physical systems, components & processes.

**PSO3:** Graduates able to work professionally on mechanical systems, thermal systems & production systems.

***Course outcome: After the completion of course students will be***

CO 1	Compute the quantitative aspects of waves and oscillations in engineering systems.
CO 2	Apply the interaction of light with matter through interference, diffraction and identify these phenomena in different natural optical processes and optical instruments.
CO 3	Analyze the behaviour of matter in the atomic and subatomic level through the principles of quantum mechanics to perceive the microscopic processes in electronic devices.
CO 5	Apply the comprehended knowledge about laser and fibre optic communication systems in various engineering applications

CO6	To differentiate holograph and photograph
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## CO VS PO'S AND PSO'S MAPPING

	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO 10	PO 11	PO 12
CO 1	3	2						1	2			1
CO 2	3	2						1	2			1
CO 3	3	2						1	2			1
CO 4	3							1	2			1
CO 5	3	2						1	2			1
CO 6	3	2						1	2			1

**Note: H-Highly correlated=3, M-Medium correlated=2, L-Less correlated=1**

### Assessment Pattern

Bloom's Category	Continuous Assessment Tests		End Semester Examination (Marks)
	Test 1 (Marks)	Test 2 (Marks)	
Remember	15	15	30
Understand	25	25	50
Apply	10	10	20
Analyse			
Evaluate			
Create			

### Mark distribution

<b>Total Marks</b>	<b>CIE MARKS</b>	<b>ESE MARKS</b>	<b>ESE Duration</b>
150	50	100	3 hours

**Continuous Internal Evaluation Pattern:**

Attendance	: 10 marks
Continuous Assessment Test (2 numbers)	: 25 marks
Assignment/Quiz/Course project	: 15 marks

**End Semester Examination Pattern:** There will be two parts; Part A and Part B. Part A contain 10 questions with 2 questions from each module, having 3 marks for each question.

Students should answer all questions. Part B contains 2 questions from each module of which student should answer any one. Each question can have maximum 2 sub-divisions and carry 14 marks

## Course Level Assessment

### Questions Course Outcome 1

#### (CO1):

1. Explain the effect of damping force on oscillators.
2. Distinguish between transverse and longitudinal waves.
3. (a) Derive an expression for the fundamental frequency of transverse vibration in a stretched string.  
(b) Calculate the fundamental frequency of a string of length 2 m weighing 6 g kept stretched by a load of 600 kg.

#### Course Outcome 2 (CO2):

1. Explain colours in thin films.
2. Distinguish between Fresnel and Fraunhofer diffraction.
3. (a) Explain the formation of Newton's rings and obtain the expression for radii of bright and dark rings in reflected system. Also explain how it is used to determine the wavelength of a monochromatic source of light.  
(b) A liquid of refractive index  $\mu$  is introduced between the lens and glass plate. What happens to the fringe system? Justify your answer.

#### Course Outcome 3 (CO3):

1. Give the physical significance of wave function?

2. What are excitons ?
3. (a) Solve Schrodinger equation for a particle in a one dimensional box and obtain its energy eigen values and normalised wave functions.  
(b) Calculate the first three energy values of an electron in a one dimensional box of width  $1 \text{ \AA}$  in electron volt.

**Course Outcome 4 (CO4):**

1. Explain reverberation and reverberation time.
2. How ultrasonic waves are used in non-destructive testing.
3. (a) With a neat diagram explain how ultrasonic waves are produced by a piezoelectric oscillator.  
(b) Calculate frequency of ultrasonic waves that can be produced by a nickel rod of length 4cm. (Young's Modulus = 207 G Pa, Density = 8900 Kg /m<sup>3</sup>)

**Course Outcome 5 (CO 5):**

1. Distinguish between spontaneous emission and stimulated emission.
2. Explain optical resonators.
3. (a) Explain the construction and working of Ruby Laser.  
(b) Calculate the numerical aperture and acceptance angle of a fibre with a core refractiveindex of 1.54 and a cladding refractive index of 1.50 when the fibre is inside water of refractive index 1.33.

**Model Question paper**

QP CODE:

PAGES:3

Reg No: \_\_\_\_\_

Name : \_\_\_\_\_

**APJ ABDUL KALAM TECHNOLOGICAL UNIVERSITY FIRST SEMESTER B.TECH  
DEGREE EXAMINATION, MONTH & YEAR**

**Course Code: PHT 110**

**Course Name: Engineering Physics B**

**Max.Marks: 100**

**Duration: 3 Hours**

**PART A**

**Answer all Questions. Each question carries 3 Marks**

1. Compare electrical and mechanical oscillators.
2. Distinguish between longitudinal and transverse waves.
3. Write a short note on antireflection coating.
4. Diffraction of light is not as evident in daily experience as that of sound waves. Give reason.
5. State and explain Heisenberg's Uncertainty principle. With the help of it explain natural line broadening.
6. Explain surface to volume ratio of nanomaterials.
7. Define sound intensity level. Give the values of threshold of hearing and threshold of pain.
8. Describe the method of non-destructive testing using ultra sonic waves
9. Explain the condition of population inversion
10. Distinguish between step index and graded index fibre. (10x3=30)

**PART B**

**Answer any one full question from each module. Each question carries 14 Marks**

**Module 1**

11. (a) Derive the differential equation of damped harmonic oscillator and deduce its solution. Discuss the cases of over damped, critically damped and under damped cases. (10)



(b) The frequency of a tuning fork is 500 Hz and its Q factor is  $7 \times 10^4$ . Find the relaxation time. Also calculate the time after which its energy becomes 1/10 of its initial undamped value. (4)

12. (a) Derive an expression for the velocity of propagation of a transverse wave in a stretched string. Deduce laws of transverse vibrations. (10)

(b) The equation of transverse vibration of a stretched string is given by  $y = 0.00327 \sin(72.1x - 2.72t)$  m, in which the numerical constants are in S.I units. Evaluate (i) Amplitude (ii) Wavelength (iii) Frequency and (iv) Velocity of the wave. (4)

### Module 2

13. (a) Explain the formation of Newton's rings and show that the radius of dark ring is proportional to the square root of natural numbers. How can we use Newton's rings experiment to determine the refractive index of a liquid? (10)

(b) Two pieces of plane glass are placed together with a piece of paper between two at one end. Find the angle of the wedge in seconds if the film is viewed with a monochromatic light of wavelength  $4800 \text{ \AA}$ . Given  $\beta = 0.0555 \text{ cm}$ . (4)

14. (a) Explain the diffraction due to a plane transmission grating. Obtain the grating equation. (10)

(b) A grating has 6000 lines per cm. Find the angular separation of the two yellow lines of mercury of wavelengths 577 nm and 579 nm in the second order. (4)

### Module 3

15. (a) Derive time dependent and independent Schrodinger equations. (10)

(b) An electron is confined to one dimensional potential box of length  $2 \text{ \AA}$ . Calculate the energies corresponding to the first and second quantum states in eV. (4)

16. (a) Classify nanomaterials based on dimensionality of quantum confinement and explain the following nanostructures. (i) nano sheets (ii) nano wires (iii) quantum dots. (10)

(b) Find the de Broglie wavelength of electron whose kinetic energy is 15 eV. (4)

### Module 4

17. (a) Explain reverberation and reverberation time? What is the significance of Reverberation time. Explain the factors affecting the acoustics of a building and their corrective measures? (10)

(b) The volume of a hall is  $3000 \text{ m}^3$ . It has a total absorption of  $100 \text{ m}^2$  sabine. If the hall is filled with audience who add another  $80 \text{ m}^2$  sabine, then find the difference in reverberation time. (4)

18. (a) With a neat diagram explain how ultrasonic waves are produced by piezoelectric oscillator. Also discuss the piezoelectric method of detection of ultrasonic waves. (10)

- (b) An ultrasonic source of 0.09 MHz sends down a pulse towards the sea bed which returns after 0.55 sec. The velocity of sound in sea water is 1800 m/s. Calculate the depth of the sea and the wavelength of the pulse. (4)

#### Module 5

19. (a) Outline the construction and working of Ruby laser. (8)  
(b) What is the principle of holography? How is a hologram recorded? (6)
20. (a) Define numerical aperture of an optic fibre and derive an expression for the NA of a stepindex fibre with a neat diagram. (10)
- (b) An optical fibre made with core of refractive index 1.5 and cladding with a fractional index difference of 0.0006. Find refractive index of cladding and numerical aperture. (4)

(14x5=70)

## SYLLABUS Engineering Physics

Course code:-PH 100

Credits:-4

Slot:-B

### Module I

Harmonic Oscillations:

Differential equation of damped harmonic oscillation, forced harmonic oscillation and their solutions  
Resonance, Q factor, Sharpness of resonance- LCR circuit as an electrical analogue of Mechanical Oscillator (Qualitative)

Waves:- One dimensional wave - differential equation and solution. Three dimensional waves - Differential equation & its solution. (No derivation) Transverse vibrations of a stretched string.

**(marks-15%)**

### Module II

Interference:- Coherence. Interference in thin films and wedge shaped films (Reflected system)  
Newton's rings measurement of wavelength and refractive index of liquid Interference filters.  
Antireflection coating.

Diffraction:- Fresnel and Fraunhofer diffraction. Fraunhofer diffraction at a single slit. Plane transmission grating. Grating equation - measurement of wavelength. Rayleigh's criterion for resolution of grating- Resolving power and dispersive power of grating. **(marks-15%)**

## **FIRST INTERNAL EXAM**

### **Module III**

Polarization of Light:-Types of polarized light. Double refraction. Nicol Prism. Quarter wave plate and half wave plate. Production and detection of circularly and elliptically polarized light. Induced birefringence- Kerr Cell - Polaroid and applications.

Superconductivity:-Superconducting phenomena. Meissner effect. Type-I and Type-II superconductors. BCS theory (qualitative). High temperature superconductors - Josephson Junction - SQUID- Applications of superconductors. **(marks-15%).**

### **Module IV**

Quantum Mechanics:-Uncertainty principle and its applications -formulation of Time dependent and Time independent Schrödinger equations- physical meaning of wave function- Energy and momentum Operators-Eigen values and functions- One dimensional infinite square well potential. Quantum mechanical Tunnelling (Qualitative)

Statistical Mechanics:-Macrostates and Microstates. Phasespace. Basic postulates of Maxwell-Boltzmann, Bose-Einstein and Fermi Dirac statistics. Distribution equations in the three cases (no derivation). Fermi Level and its significance. **(marks-15%)**

## **SECOND INTERNAL EXAM**

### **Module V**

Acoustics:-Intensity of sound- Loudness-Absorption coefficient - Reverberation and reverberation time- Significance of reverberation timeSabine's formula (No derivation) -Factors affecting acoustics of a building.

Ultrasonics:-Production of ultrasonic waves - Magnetostriction effect and Piezoelectric effect - Magnetostriction oscillator and Piezoelectric oscillator - Detection of ultrasonics - Thermal and piezoelectric methods-Applications of ultrasonics - NDT and medical. **(marks-20%)**

### **Module VI**

Laser:-Properties of Lasers, absorption, spontaneous and stimulated emissions, Population inversion, Einstein's coefficients, Working principle of laser,Optical resonant cavity.Ruby Laser, Helium-Neon Laser, Semiconductor Laser (qualitative). Applications of laser, holography (Recording and reconstruction)

Photonics:-Basics of solid state lighting - LED – Photodetectors - photo voltaic cell, junction and avalanche photo diodes, photo transistors, thermal detectors, Solar cells- I-V characteristics - Optic fibre-Principle of propagation-numerical aperture-optic communication system (block diagram) - Industrial, medical and technological applications of optical fibre.Fibre optic sensors - Basics of Intensity modulated and phase modulated sensors.

**(marks-20%)**

#### **Text Books:-**

- Aruldhas, G., Engineering Physics, PHI Ltd.
- Beiser, A., Concepts of Modern Physics, McGraw Hill India Ltd.
- Bhattacharya and Tandon, Engineering Physics , Oxford India
- Brijlal and Subramanyam, A Text Book of Optics, S. Chand Co.
- Dominic and Nahari, A Text Book of Engineering Physics, Owl Books Publishers

- Hecht, E., Optics, Pearson Education
- Mehta, N., Applied Physics for Engineers, PHI Ltd
- Palais, J. C., Fiber Optic Communications, Pearson Education
- Pandey, B. K. and Chaturvedi, S., Engineering Physics, Cengage Learning
- Philip, J., A Text Book of Engineering Physics, Educational Publishers
- Premlet, B., Engineering Physics, Mc GrawHill India Ltd
- Sarin, A. and Rewal, A., Engineering Physics, Wiley India Pvt Ltd
- Sears and Zemansky, University Physics , Pearson
- Vasudeva, A. S., A Text Book of Engineering Physics, S. Chand Co

## QUESTION BANK

### Module – I

Q.No	Questions	CO	KL
1	What do you mean by oscillation?	CO1	K1
2	Explain angular frequency?	CO1	K2
3	Define damped oscillation and forced oscillation	CO1	K2
4	Derive the differential equation of SHM	CO1	K3
5	Derive forced harmonic oscillation	CO1	K3
6	What do you mean by resonance and sharpness of resonance ?	CO1	K1
7	Compare electrical and mechanical oscillation	CO1	K2
8	A transverse wave on a stretched string is described by $Y(x,y)=4.0\sin(25t+0.016x+\pi/3)$ where x and y are in CM and t is in second obtain a) speed b) amplitude c) frequency d) initial phase of origin	CO1	K4
9	State the transverse vibrations of a stretched string	CO1	K2

10	A piece of wire 50 cm long is stretched by a load of 2.5kg and has a mass of 1.44kg. Find the frequency of the second harmonic?	CO1	K4
11	Calculate the speed of transverse wave in a string of cross sectional area $1\text{mm}^2$ under tension of 1kg wt density of wire $=10.5 \times 10^3\text{kg/m}^3$	CO1	K4

## Module – II

Q.No	Questions	CO	KL
1	State the conditions for sustained interference	CO2	K2
2	Explain the term coherent source of light	CO2	K1
3	What is diffraction grating?	CO2	K1
4	Derive the relation for $n^{\text{th}}$ diameter ring of newton's ring .Why rings are closer for higher order?	CO2	K3
5	State Rayleigh criterion for resolving power	CO2	K1
6	State the difference between diffraction and interference	CO2	K1
7	Explain fraunhofer diffraction through a single slit	CO2	K1
8	What is interference and derive the equation for interference on a thin film ?	CO2	K1
9	Derive the equation for wedge shaped film and explain it	CO2	K2
10	Differentiate between fresnel and fraunhofer diffraction	CO2	K3
11	Explain newton's ring and derive its equation	CO2	K1

## Module – III

Q.No	Questions	CO	KL
1	Explain the construction and working of nicol prism	CO3	K1

2	Explain how a quarter wave plate is used for producing circularly polarized light	CO3	K1
3	Explain dc and ac Josephson effect	CO3	K1
4	Distinguish between soft and hard type conductors	CO3	K2
5	Mention any three applications of superconductors	CO3	K1
6	Explain about SQUID	CO3	K1
7	Explain salient features of BCS theory	CO3	K1
8	Explain meissner effect	CO3	K1
9	Explain high temperature superconductivity	CO3	K1
10	Explain the production and detection of circularly and elliptically polarized light	CO3	K3
11	Explain the polarization phenomena? What are the types of polarized light and its application?	CO3	K4

### Module – IV

Q.No	Questions	CO	KL
1	Explain eigen values and eigen functions	CO4	K1
2	What are matter waves ? write the wave function for matter wave	CO4	K3
3	Explain tunneling in quantum mechanics	CO4	K1
4	Write the physical meaning of a wave function	CO4	K2
5	State Heisenberg's uncertainty principle	CO4	K2
6	Calculate de Broglie wavelength of an electron whose kinetic energy is 10keV	CO4	K4
7	Electrons cannot be occupied inside the nucleus .Justify the statement with proof	CO4	K2
8	State Heisenberg's uncertainty principle. Explain non occurrence of	CO4	K2

	electron with in nucleus		
9	Obtain schrodinger's time dependent equation	CO4	K2
10	An electron and proton has the same non relativistic KE which one has lesser wavelength? Why?	CO4	K3
11	Write down the schrodinger's equation for a particle in one dimensional square well potential .solve the same to obtain its energy eigen values	CO4	K3

### Module – V

Q.No	Questions	CO	KL
1	What do you mean by acoustics?	CO5	K1
2	Explain loudness and units of loudness	CO5	K3
3	Explain loudness and units of loudness	CO5	K1
4	What is absorption and absorption coefficient?	CO5	K1
5	What do you mean by reverberation? Explain reasons for it	CO5	K3
6	What is reverberation time?	CO5	K1
7	Explain sabine's formula	CO5	K2
8	What are the factors affecting acoustics of a building and their remedies?	CO5	K2
9	Write the properties of ultrasonic waves	CO5	K2
10	Explain the applications of ultrasonic's	CO5	K3
11	Explain hoe piezoelectric effect is utilized for the production of ultrasonic waves .Explain some of the applications of ultrasonics	CO5	K4

### Module – VI



<b>Q.No</b>	<b>Questions</b>	<b>CO</b>	<b>KL</b>
1	Name four outstanding characteristics of laser	CO6	K2
2	What is population inversion?	CO6	K2
3	What is LED? Define its working principal.	CO6	K3
4	Explain the principle of working for avalanche photo diode	CO6	K2
5	What is the principle of holography? write its applications	CO6	K3
6	Draw and explain V-I characteristics of a photo transistor	CO6	K2
7	Explain principle of propagation of light through an optic fiber	CO6	K2
8	Distinguish between step index fibre and graded index fibre	CO6	K3
9	What are photovoltaic cells?	CO6	K2
10	Explain with necessary theory the working of any four level laser	CO6	K2
11	Write any two advantages of hologram over photographic images	CO6	K3

# Module - I

## Chapter - I

### Oscillations

#### Harmonic Motion

The displacement of the particle executing oscillatory motion that can be expressed in terms of sine or cosine functions are known as Harmonic motion. The simplest type of harmonic motion is called Simple Harmonic motion (SHM).

#### Periodic Motion

A motion which repeats itself ~~at~~ after regular intervals of time  $T$  called periodic motion.

Eg: Oscillations of simple pendulum

motion of Earth around sun etc...

#### Oscillatory Motion

A motion in which a particle moves to and fro about a fixed point and repeats the motion after a regular intervals of time  $T$  is called oscillatory motion.

Eg: Oscillations of simple pendulum and loaded spring

## Simple Harmonic Motion

A particle is said to execute simple harmonic motion if it moves to and fro periodically along a path such that the restoring force acting on it is proportional to its displacement from a fixed point and is always directed towards that point.

Differential equation for SHM

Consider a particle of mass  $m$  executing SHM along a straight line.

Then  $F \propto \text{displacement}$

$$F \propto -x$$

$$F = -kx$$

where  $k$  is the proportionality constant or spring constant. The -ve sign indicates that the restoring force acts against displacement.

$$\begin{aligned} \text{ie } F &= -kx \\ ma &= -kx \\ m \frac{d^2x}{dt^2} &= -kx \end{aligned} \left\{ \begin{aligned} a &= \frac{dv}{dt} = \frac{d}{dt} \left( \frac{dx}{dt} \right) \\ &= \frac{d^2x}{dt^2} \end{aligned} \right.$$

$$\frac{m d^2x}{dt^2} + kx = 0 \Rightarrow \text{differential eqn for SHM}$$

OR

$$\frac{d^2x}{dt^2} + \frac{k}{m}x = 0$$

$$\frac{d^2x}{dt^2} + \omega^2x = 0 \quad \text{--- (1)}$$

Multiplying above eqn by  $2\frac{dx}{dt}$

$$2\frac{dx}{dt} \frac{d^2x}{dt^2} + 2\frac{dx}{dt} \omega^2x = 0 \quad \text{--- (2)}$$

Then eqn (2) can be written as

$$\frac{d}{dt} \left( \left( \frac{dx}{dt} \right)^2 + \omega^2x^2 \right) = 0$$

Now integrating  $\left( \frac{dx}{dt} \right)^2 + \omega^2x^2 = C \quad \text{--- (3)}$

where  $C$  is a constant of integration

To find  $C$

The velocity of the particle at the external position is zero. If ' $a$ ' is the maximum amplitude (maximum displacement), then

$$\frac{dx}{dt} = 0 \quad \text{at } x = a$$

Substitute this in eqn (3)

$$C = \omega^2a^2$$

Then put  $C = \omega^2a^2$  in eqn (3)

$$\left(\frac{dn}{dt}\right)^2 + \omega^2 n^2 = \omega^2 a^2$$

$$\left(\frac{dn}{dt}\right)^2 = \omega^2 a^2 - \omega^2 n^2$$

$$\left(\frac{dn}{dt}\right)^2 = \omega^2 (a^2 - n^2)$$

$$\frac{dn}{dt} = \omega \sqrt{a^2 - n^2} \quad \text{--- (4)}$$

$$\frac{dn}{dt} \text{ is Velocity } v = \omega \sqrt{a^2 - n^2} \quad \text{--- (5)}$$

$$\text{From eqn (4)} \quad \frac{dn}{dt} = \omega \sqrt{a^2 - n^2}$$

$$\frac{dn}{\sqrt{a^2 - n^2}} = \omega dt$$

$$\text{Then integrating } \sin^{-1}\left(\frac{n}{a}\right) = \omega t + \phi$$

where  $\phi$  is const of integration

$$\text{i.e., } \frac{n}{a} = \sin(\omega t + \phi) \quad \text{or } n = a \sin(\omega t + \phi) \quad \text{--- (6)}$$

$n$  is the displacement of the particle at any instant  $t$  and  $(\omega t + \phi)$  is the phase of oscillation at any instant

$\Rightarrow$  Now, the initial phase  $\phi = \delta + \frac{\pi}{2}$

$$\text{The } n = a \sin(\omega t + \delta + \frac{\pi}{2})$$

$$n = a \cos(\omega t + \delta) \quad \text{--- (7)}$$

also represents SHM  
if  $t$  is increased by  $\frac{2\pi}{\omega}$

$$m = a \sin \left( \omega t + \frac{2\pi}{\omega} \right) + \phi$$

$$= a \sin (\omega t + 2\pi + \phi) = a \sin (\omega t + \phi)$$

$\therefore$  The eqn repeats itself after a time  
 $\frac{2\pi}{\omega}$ ,  $4\pi/\omega$  etc

Hence  $\frac{2\pi}{\omega}$  is called the period or  $T = \frac{2\pi}{\omega}$

$$\omega = \sqrt{\frac{k}{m}} \quad \text{or} \quad T = 2\pi \sqrt{\frac{m}{k}}$$

## Damped Harmonic Oscillation

In Free Oscillations total energy of the system remains constant.

The decrease in amplitude of an oscillation caused by dissipative forces is called damping.

In real situations, the total energy is dissipated to its surroundings and the amplitude decays to zero.

Damped Harmonic Oscillator.

When a medium particle in a medium oscillates, a damping force acts in the particle and gradually decrease the amplitude, such as

oscillator is called damped harmonic oscillator and the corresponding motion is called Damped Harmonic Oscillation.

### Differential Equation of Damped Harmonic Oscillator

Consider a particle executing damped harmonic oscillation in a medium. The forces acting on it are

- i) Restoring force =  $-kx$
- ii) Damping Force =  $-b \frac{dx}{dt}$  where  $b$  is called damping constant.

Then  $F = f_1 + f_2$

$$m \frac{d^2x}{dt^2} = -kx - b \frac{dx}{dt}$$

$$m \frac{d^2x}{dt^2} + b \frac{dx}{dt} + kx = 0$$

$$m \left\{ \frac{d^2x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + \frac{k}{m} x \right\} = 0$$

$$\frac{d^2x}{dt^2} + \frac{b}{m} \frac{dx}{dt} + \frac{k}{m} x = 0$$

Put  $\frac{b}{m} = 2\gamma$ , where  $\gamma$  is damping coefficient

$\frac{k}{m} = \omega_0^2$ , where  $\omega_0$  is the natural angular

frequency of the oscillation in the absence of damping force

$$\text{Then } \frac{d^2m}{dt^2} + 2r \frac{dm}{dt} + \omega_0^2 m = 0 \quad \text{--- (2)}$$

This is the differential equation of damped harmonic oscillator.

Solution of the equation

Assume the solution of the form  $m = Ae^{nt}$

Then differentiating  $\frac{dm}{dt} = Ane^{nt} = \alpha n$

$$\frac{d^2m}{dt^2} = \alpha^2 Ae^{nt} = d^2m$$

Substitute the values in eqn (2)

$$\alpha^2 m + 2r \alpha n + \omega_0^2 m = 0$$

$$d^2 + 2rd + \omega_0^2 = 0$$

The roots of the eqn  $d = \frac{-2r \pm \sqrt{4r^2 - 4\omega_0^2}}{2}$

$$\text{Then } m = Ae^{(-r \pm \sqrt{r^2 - \omega_0^2})t}$$

ie, the solutions.  $m_1 = A_1 e^{(-r + \sqrt{r^2 - \omega_0^2})t}$

$$m_2 = A_2 e^{(-r - \sqrt{r^2 - \omega_0^2})t}$$

Where  $A_1$  &  $A_2$  are constants which depends on the initial values of position and velocity the value of 'r' determines the behavior of the system.



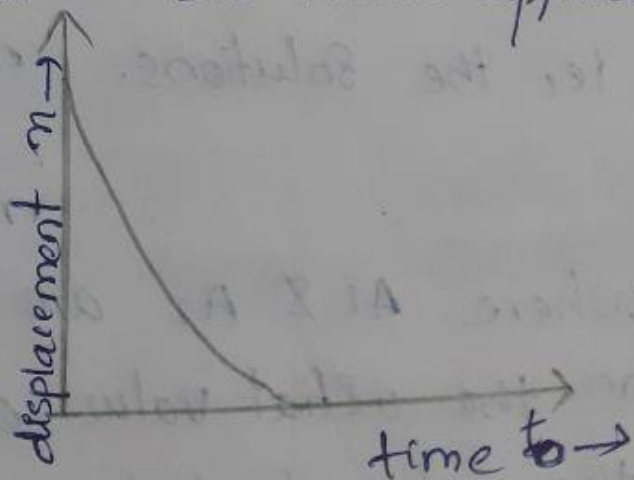
The general solution is

$$x = A_1 e^{(-\gamma + \sqrt{\gamma^2 - \omega_0^2})t} + A_2 e^{(-\gamma - \sqrt{\gamma^2 - \omega_0^2})t} \quad (3)$$

case I Over damped case ( $\gamma > \omega_0$ )

If the damping is so high such that  $\gamma > \omega_0$  then  $\sqrt{\gamma^2 - \omega_0^2}$  is a real quantity and  $\sqrt{\gamma^2 - \omega_0^2}$  is less than  $\gamma$ . Thus  $(-\gamma + \sqrt{\gamma^2 - \omega_0^2})t$  &  $(-\gamma - \sqrt{\gamma^2 - \omega_0^2})t$  are both  $-ve$ . So the displacement ( $x$ ) decays exponentially to zero without any oscillation. This motion is called over damped or dead Beat or Aperiodic.

Aperiodic - The particle when once displaced returns to equilibrium position slowly without performing any oscillation. Its main application is in Dead beat



case ② - Critically damped ( $r = \omega_0$ ).

Applying the condition in eqn ③

Then  $\sqrt{r^2 - \omega_0^2} = 0$  Or general soln will be

$$m = A_1 e^{-rt} + A_2 e^{-rt} = (A_1 + A_2) e^{-rt}$$

$$\text{let } A_1 + A_2 = C, \text{ Then } m = C e^{-rt}$$

In this eqn there is only one constant and ~~there~~ hence does not form the solution by the second order differential equation.

$$\therefore \sqrt{r^2 - \omega_0^2} = b$$

Then eqn ③ becomes

$$m = A_1 e^{-rt+bt} + A_2 e^{-rt-bt}$$

$$= A_1 e^{-rt} e^{bt} + A_2 e^{-rt} e^{-bt}$$

$$= e^{-rt} (A_1 e^{bt} + A_2 e^{-bt})$$

$$= e^{-rt} \left\{ A_1 \left( 1 + bt + \frac{(bt)^2}{2} + \dots \right) + A_2 \left( 1 - bt + \frac{(bt)^2}{2} - \dots \right) \right\}$$

• Neglecting higher process if  $b$  due to its

Small magnitude

$$m = e^{-rt} \{ A_1 + A_1 bt + A_2 - A_2 bt \}$$

$$= e^{-rt} \{ (A_1 + A_2) + (A_1 - A_2) bt \}$$

$$\text{put } A_1 + A_2 = P \quad \& \quad (A_1 - A_2)h = \phi$$

$$\text{Then } m = e^{-\gamma t} \{ P + \phi t \} \quad \text{--- (4)}$$

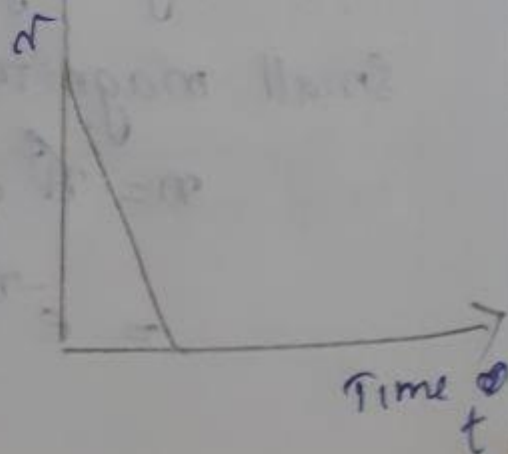
From the above eqn initially as  $t$  increases  $P + \phi t$  increase and the displacement also increase out as the time ~~to~~ increases the exponential term increases more than  $(P + \phi t)$  term. Then the displacement decreases from maximum value to zero quickly. The motion neither damped nor oscillatory. This motion is called ~~over~~ critically damped or just oscillatory. ~~This motion is called~~ here the particle acquires the position of equilibrium very rapidly.

Applications  $\leftarrow$  pointer type instruments like galvanometer where the pointer moves at once to have a correct position and stay at this position without any ~~an~~ oscillation.

$\Rightarrow$  Automobile shock absorbers

$\Rightarrow$  Door close mechanisms

$\Rightarrow$  Recoil mechanism in guns.



case ③ under damped case ( $r < \omega_0$ )

Here  $\sqrt{r^2 - \omega^2}$  is imaginary

$$\sqrt{r^2 - \omega^2} = i\omega = i\sqrt{\omega_0^2 - r^2}$$

Then eqn ③ will be

$$x = A_1 e^{(-r + i\omega)t} + A_2 e^{(-r - i\omega)t}$$

$$x = e^{-rt} (A_1 e^{i\omega t} + A_2 e^{-i\omega t})$$

$$= e^{-rt} \{ A_1 (\cos \omega t + i \sin \omega t) + A_2 (\cos \omega t - i \sin \omega t) \}$$

$$x = e^{-rt} \{ A_1 + A_2 (\cos \omega t) + i(A_1 - A_2) \sin \omega t \}$$

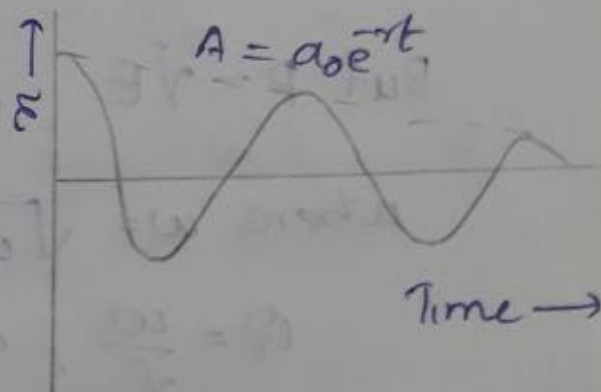
put  $A_1 + A_2 = a_0 \sin \phi$  &  $i(A_1 - A_2) = a_0 \cos \phi$   
ie  $x = a_0 e^{-rt} (\sin \phi \cos \omega t + \sin \omega t \cos \phi)$

$$x = a_0 e^{-rt} \sin(\omega t + \phi) \quad \text{--- ⑤}$$

eqn ⑤ shows that motion is oscillatory. The amplitude  $a_0 e^{-rt}$  is not a constant but decreases with time

Applications

⇒ Ballistic Galvanometer



effect of damping

1. the amplitude of oscillation decreases exponentially with time.
2. the frequency of oscillation of a damped oscillator is less than the frequency of undamped oscillations.

### Quality Factor

Quality factor is defined as  $2\pi$  times the ratio of energy stored to the energy loss per period.

$$Q = 2\pi \frac{\text{energy stored}}{\text{energy loss per period}}$$
$$= \frac{2\pi E}{PT}$$

$$\left\{ \begin{aligned} Q &= \frac{2\pi E}{-\frac{dE}{dt} \times t} = 2\pi \frac{E}{PT} \quad P = \text{power dissipation} \\ &= \frac{-dE}{dt} \end{aligned} \right.$$

But  $P = \sqrt{E}$   $\therefore Q = \frac{2\pi E}{\sqrt{E} T} \Rightarrow \frac{2\pi}{\sqrt{T}} = \frac{2\pi}{\sqrt{\left(\frac{2\pi}{\omega}\right)}}$

where  $\omega = \sqrt{\omega_0^2 - \gamma^2}$

$$Q = \frac{\omega}{\gamma}, \quad \gamma = \frac{b}{2m}, \quad b \text{ is damping const}$$

Then  $Q = \frac{2\omega m}{b}$   $\&$   $Q$  is dimensionless

## Forced or Driven Harmonic Oscillations

If an external periodic force is applied on a damped harmonic oscillator, the oscillatory system is called driven or forced harmonic oscillator.

or

An oscillator which is forced to oscillate with a frequency other than its natural frequency is known as forced or driven harmonic oscillator.

The forces acting on a forced oscillator are

- 1) Restoring force  $-kx$
- 2) The damping force  $-bV$
- 3) External driving periodic force  $F_0 \sin \omega_f t$ , where  $F_0$  is amplitude

$$\therefore F = F_1 + F_2 + F_3$$

$$ma = -kx - bV + F_0 \sin \omega_f t$$

$$m \frac{d^2x}{dt^2} = -kx - bV + F_0 \sin \omega_f t \quad \text{--- ①}$$

$$\frac{d^2x}{dt^2} + \frac{kx}{m} + \frac{b}{m} V = \frac{F_0}{m} \sin \omega_f t \quad \text{--- ②}$$

$$\text{but } V = \frac{dx}{dt}$$

Then eqn ② becomes

$$\frac{d^2m}{dt^2} + \frac{k}{m}m + \frac{b}{m}\frac{dm}{dt} = f_0 \sin \omega_f t \quad \text{--- ③}$$

where  $\sqrt{k/m} = \omega_0$ , The natural frequency of the body and  $\frac{b}{m} = 2d$ , the damping constant for unit mass &  $\frac{f_0}{m} = f_0$

$$\text{Then } \frac{d^2m}{dt^2} + 2d\frac{dm}{dt} + \omega_0^2 m = f_0 \sin \omega_f t \quad \text{--- ④}$$

above eqn represent differential eqn for forced harmonic oscillator.

③ Solution.

$$m = A \sin(\omega_f t - 0) \quad \text{--- ⑤}$$

$$\frac{dm}{dt} = A \omega_f \cos(\omega_f t - 0)$$

$$\frac{d^2m}{dt^2} = -A \omega_f^2 \sin(\omega_f t - 0)$$

Sub this in eqn ④

$$\begin{aligned} -A \omega_f^2 \sin(\omega_f t - 0) + 2d A \omega_f \cos(\omega_f t - 0) + \omega_0^2 A \sin(\omega_f t - 0) \\ = f_0 \sin(\omega_f t - 0) \end{aligned}$$

(In RHS, we added & subtracted 0)

$$\text{ie, } -A\omega_f^2 \sin(\omega_f t - \theta) + 2r A\omega_f \cos(\omega_f t - \theta) + \omega_0^2 A \sin(\omega_f t - \theta) = f_0 (\sin(\omega_f t - \theta) \cos \theta + \cos(\omega_f t - \theta) \sin \theta)$$

Taking like terms we get

$$(-A\omega_f^2 - f_0 \cos \theta + \omega_0^2 A) \sin(\omega_f t - \theta) + (2r A\omega_f - f_0 \sin \theta) \cos(\omega_f t - \theta) = 0 \quad \text{--- (8)}$$

To find A

equating the coefficients of  $\sin(\omega_f t - \theta)$  &  $\cos(\omega_f t - \theta)$ , which are zero seperating

$$\therefore -A\omega_f^2 - f_0 \cos \theta + \omega_0^2 A = 0$$

$$-A\omega_f^2 + \omega_0^2 A = f_0 \cos \theta \quad \text{--- (8)}$$

$$2r A\omega_f - f_0 \sin \theta = 0$$

$$2r A\omega_f = f_0 \sin \theta \quad \text{--- (9)}$$

Squaring and adding (8) & (9) we get

$$(-A\omega_f^2 + \omega_0^2 A)^2 + 4r^2 A^2 \omega_f^2 = f_0^2$$

$$A^2 \{ (\omega_0^2 - \omega_f^2)^2 + 4r^2 \omega_f^2 \} = f_0^2$$

$$A = \frac{f_0}{\sqrt{(\omega_0^2 - \omega_f^2)^2 + 4r^2 \omega_f^2}} \quad \text{--- (10)}$$



which is the amplitude of force oscillation.

Phase difference

Dividing eqn ④ by ③

$$\tan \theta = \frac{2rA\omega_f}{A(\omega_0^2 - \omega_f^2)} = \frac{2r\omega_f}{\omega_0^2 - \omega_f^2} \quad \text{--- ⑪}$$

This gives the phase difference b/w forced oscillation & applied force

Sub. for A in eqn ⑤

$$x = \frac{f_0 \sin(\omega_f t - \theta)}{\sqrt{(\omega_0^2 - \omega_f^2)^2 + 4r^2\omega_f^2}} \quad \text{--- ⑫}$$

Above eqn shows that the system vibrate with the frequency of the applied periodic force and having a phase difference of  $\theta$

Case I Low driving frequency  $\omega_f < \omega_0$

$$A = \frac{f_0}{\sqrt{(\omega_0^2 - \omega_f^2)^2 + 4r^2\omega_f^2}}$$

neglecting  $\omega_f^2$ , since  $\omega_f$  is less than  $\omega_0$

$$A = \frac{f_0}{\omega_0^2} = \frac{f_0/m}{k/m} = \frac{f_0}{k}$$

Amplitude ~~do~~ not depend on mass of oscillating body

Case II ( $\omega_f = \omega_0$ ) Resonance

Resonance is a phenomenon that occurs when a vibrating system or external force drives another system to oscillate with greater amplitude at a specific frequency

Here  $\omega_f = \omega_0$

$$\therefore A = \frac{f_0}{2r\omega_f} \quad \& \quad \tan\theta = \frac{2r\omega_f}{\omega_0^2 - \omega_f^2} = \infty$$

or  $\theta = \frac{\pi}{2}$

Case III

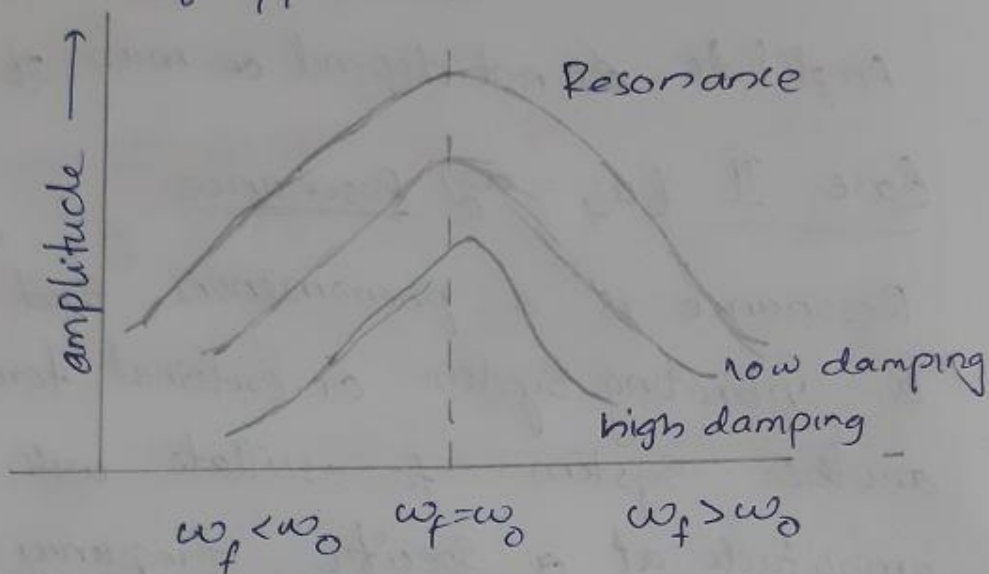
High Driving Frequency  $\omega_f > \omega_0$

$$A = \frac{f_0}{\sqrt{(\omega_0^2 - \omega_f^2)^2 + 4r^2\omega_f^2}}$$

when  $\omega_f > \omega_0$

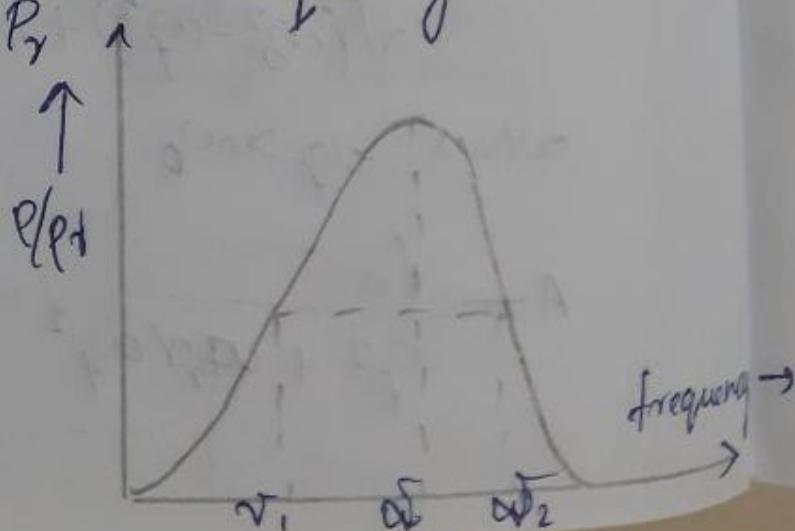
$$A \approx \frac{f_0}{\omega_f^2 + r\omega_f^2} = \frac{f_0}{\omega_f^2} \text{ for low clamping}$$

Variation Of Amplitude  $A$  with frequency  $\omega_f$  of applied force



### Sharpness Of Resonance

The rate of change (fall) of amplitude with the change of frequency of the applied periodic force on either side of resonant frequency is known as sharpness of resonance. Let  $P_r$  is the power absorbed at resonance,  $P$  is the power absorbed at any frequency  $\nu$  a graph is drawn between  $\frac{P}{P_r}$  & frequency



## LCR Circuit as Electrical analogue of Mechanical Oscillators.

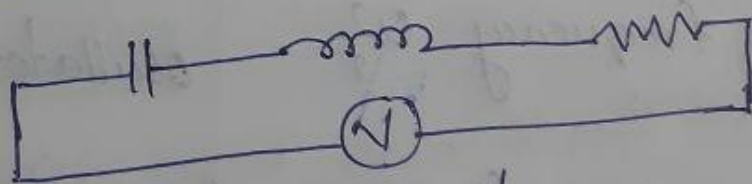
### Oscillations in an LC Circuit.

A pure LC circuit is an electrical analogue of the simple pendulum. In the case of simple pendulum energy alternates between the ~~potential~~ potential and kinetic energy. In case of LC circuit energy is alternately shared in the capacitor as electric field and in inductor as magnetic field.

In LC circuit frequency of oscillation

$$n = \frac{1}{2\pi\sqrt{LC}}$$

### Forced Oscillation in A Series LCR Circuit



$$V = V_0 \sin \omega t$$

Applying Kirchoff's Voltage law to the circuit

$$V_L + IR + V_C = V_0 \sin \omega t$$

$$L \frac{di}{dt} + IR + \frac{Q}{C} = V_0 \sin \omega t$$

$$L \frac{d^2q}{dt^2} + iR + \frac{q}{c} = V_0 \sin \omega t$$

$$\text{ie, } \frac{d^2q}{dt^2} + \frac{R}{L} \frac{dq}{dt} + \frac{q}{c} = \frac{V_0}{L} \sin \omega t$$

This is the differential equation in case of Forced Oscillation.

Mechanical Oscillator	Electrical Oscillator
Displacement $x$	charge $q$
Velocity $\frac{dx}{dt}$	Current $\frac{dq}{dt}$
mass $m$	Inductance $L$
damping coefficient $\gamma$	Resistance $R$
Force amplitude $F_0$	voltage amplitude $V_0$
Driving frequency $\omega_f$	oscillator frequency $\omega_0$

The angular frequency of damped oscillations in LCR circuit is given by

$$\omega = \sqrt{\frac{1}{LC} - \frac{R^2}{4L^2}}$$

# Waves

## Wave Motion.

wave is a form of disturbance which propagate through space. It transfers energy from one ~~part~~ region of space to another region without transferring matter along with.

## Mechanical Waves

Waves which require a medium for their propagation are known as mechanical waves.

## Electromagnetic Waves

Waves which do not require a medium for their propagation are known as E.M waves

## Progressive Waves

A wave which travel onward with the transfer of energy across any medium is known as progressive wave it is ~~known~~ moving continuously along the same direction.

## Stationary Wave.

The progressive waves travelling through the same medium in opposite direction form a stationary or standing wave. Stationary wave do not transfer energy from one place to another. The crest & trough and zone of compression & rare fraction merely appear and disappear in fixed positions.

### wavelength

→ The distance b/w two consecutive crests or troughs is called wavelength by transverse wave

⇒ ~~Wavelength~~ <sup>or</sup> wavelength is also defined as the distance travelled by the wave during the time a particle of the medium complete  $n$  one vibration about its mean position. It is denoted by  $\lambda$

$$\text{ie, } v = \lambda \nu \quad \text{or} \quad \lambda = \frac{v}{\nu}$$

### Transverse Wave Motion

When the particle of the medium vibrate about their mean position in a direction perpendicular to the direction of propagation of a wave, it is called a transverse wave

Eg: Light wave, waves produced in a string under tension

## Longitudinal wave motion

When the particles of the medium vibrate about their mean position parallel to the direction of the propagation of waves it is called a longitudinal wave.

Eg: Sound waves etc.

The distance b/w two consecutive compressions or rarefactions is called wavelength of longitudinal wave.

## General equation of wave motion.

### one dimensional waves

waves travelling along a line or axis is known as one dimensional wave.

Eg: waves through a string or through a spring

Consider a wave pulse moves in a direction with a velocity  $v$  after a time  $t$  the pulse has moved a distance  $vt$ .

let  $u(x, t)$  be transverse displacement at  $x$ , which is a fn of  $x$  &  $t$

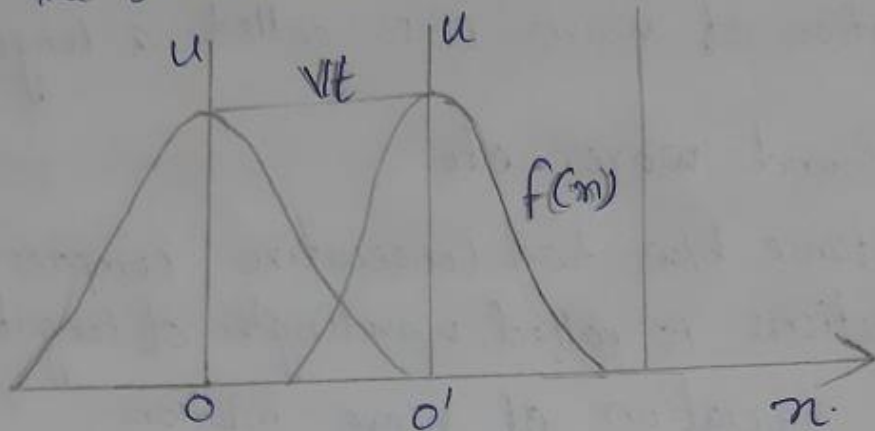
$$\text{i.e., } u(x, t) = f(x, t)$$

$$\text{At } x=0, u(x, 0) = f(x, 0)$$

when  $f$  describes the shape of wave function. after a time  $t$  the pulse travelled a distance



$\forall t$  since the shape of the wave does not change as it travels the wave form must be represented by the same wave function.



Then  $n' = n - vt$

or  $u(n, t) = f(n - vt)$

if the pulse moving in opposite direction

$u(n, t) = f(n + vt)$

### Sinusoidal waves

Consider a transverse wave having a sinusoidal shape at  $t=0$  i.e.

$u(n, 0) = f(n, 0) = a \sin \omega t = a \sin \frac{2\pi}{\lambda} n$

if the wave travels ~~to~~ with a velocity  $v$  in the direction of  $n$  axis

$u(n, t) = \sin \left( a \sin \frac{2\pi}{\lambda} (n - vt) \right)$

$\frac{2\pi}{\lambda} (n - vt)$  is called phase of wave

at time  $t$

$$u = a \sin \frac{2\pi}{\lambda} (x - vt)$$

$$= a \sin \left( \frac{2\pi}{\lambda} x - \frac{2\pi}{\lambda} vt \right)$$

$$\Rightarrow u = \sin(kx - \omega t)$$

Particle Velocity And wave Velocity

particle velocity is the velocity of the particle of the motion undergoing SHM when a harmonic wave travels through it

$$v_p = \frac{dy}{dt}$$

wave velocity:

wave velocity is the velocity of the wave moving in a direction for a wave frequency with a perpendicular ~~force~~ phase.

Differentiating, ~~dm~~  $dm - v dt = 0$

$$\text{or } v = \frac{dm}{dt}$$

General wave Equation

1D wave equation

The equation of wave motion is given by

$$u = f(x - vt) \quad \text{--- (1)}$$

$$\begin{aligned}
 u &= a \sin \frac{2\pi}{\lambda} (x - vt) \\
 &= a \sin \left( \frac{2\pi}{\lambda} x - \frac{2\pi}{\lambda} vt \right)
 \end{aligned}$$

$$u = \sin(kx - \omega t)$$

Particle Velocity And wave Velocity

particle velocity is the velocity of the particle of the motion undergoing SHM when a harmonic wave travels through it

$$v_p = \frac{dy}{dt}$$

wave velocity:

wave velocity is the velocity of the wave moving in a direction for a wave frequency with a perpendicular ~~force~~ phase.

Differentiating, ~~dm~~  $dm - v dt = 0$

$$\text{or } v = \frac{dm}{dt}$$

General wave Equation

1D wave equation

The equation of wave motion is given by

$$u = f(x - vt) \quad \text{--- (1)}$$

Differentiating eqn ① w.r.t  $x$  twice

$$\frac{du}{dx} = f'(x-vt) \quad \text{--- ②}$$

$$\frac{d^2u}{dx^2} = f''(x-vt) \quad \text{--- ③}$$

Differentiating eqn ① w.r.t  $t$  twice

$$\frac{du}{dt} = f'(x-vt) \cdot v \quad \text{--- ④}$$

$$\frac{d^2u}{dt^2} = v^2 f''(x-vt) \quad \text{--- ⑤}$$

Sub eqn ③ in ⑤ we get

$$\frac{d^2u}{dt^2} = v^2 \frac{d^2u}{dx^2} \quad \text{or} \quad \frac{d^2u}{dx^2} = \frac{1}{v^2} \frac{d^2u}{dt^2} \quad \text{--- ⑥}$$

This is called 1D differential eqn of wave motion

From eqn ② & ④  $\frac{du}{dt} = v \frac{du}{dx}$

$\frac{du}{dt} \Rightarrow$  Particle velocity

$v \Rightarrow$  wave velocity &  $\frac{du}{dx} \Rightarrow$  slope of my wave  
ie, Particle velocity = wave velocity  $\times$  slope of my wave

Solution

Solution in the form  $\frac{d^2u}{dx^2} = \frac{1}{v^2} \frac{d^2u}{dt^2} \quad \text{--- ①}$

$$u(x,t) = x(x)T(t) \quad \text{--- ②}$$

$x(x)$  is a fn of  $x$  &  $T(t)$  is a fn of  $t$

Differentiating ① twice w.r.t  $n$  & w.r.t  $t$   
and substitute in eqn ②

$$\frac{du}{dn} = \frac{dx}{dn} T \quad \& \quad \frac{du}{dt} = x \frac{dT}{dt}$$

$$\frac{d^2u}{dn^2} = \frac{d^2x}{dn^2} T \quad \frac{d^2u}{dt^2} = x \frac{d^2T}{dt^2}$$

$$\text{ie } T \frac{d^2x}{dn^2} = \frac{x}{\sqrt{2}} \frac{d^2T}{dt^2} \quad \text{--- ③}$$

dividing eqn ③ by  $xT$

$$\frac{1}{x} \frac{d^2x}{dn^2} = \frac{1}{\sqrt{2}} \frac{1}{T} \frac{d^2T}{dt^2} \quad \text{--- ④}$$

$$\frac{1}{x} \frac{d^2x}{dn^2} = -k^2 \quad \& \quad \frac{d^2x}{dn^2} = -k^2 x \quad \text{--- ⑤}$$

$$\text{Similarly } \frac{d^2T}{dt^2} = k^2 \sqrt{2} T \quad \text{--- ⑥}$$

eqn ⑤ & ⑥ are 2<sup>nd</sup> order differential equations & their solutions can be written in terms of exponential

forms

$$\text{ie, } x(n) = C e^{\pm ikn} \quad \text{--- ⑦}$$

$$T(t) = C e^{\pm i\omega t} \quad \text{--- ⑧}$$

Combining these,  $u(x, t) = C e^{i(kx \pm \omega t)}$

$$u(x, t) = C e^{i(kx \pm \omega t)} \quad \text{or}$$

$C$  is a constant & can be found by initial condition.

### 3 Dimensional wave equation

In 3 Dimension the wave eqn can be written as

$$\frac{d^2 u}{dx^2} + \frac{d^2 u}{dy^2} + \frac{d^2 u}{dz^2} = \frac{1}{v^2} \frac{d^2 u}{dt^2} \quad \text{or}$$

$$\nabla^2 u = \frac{1}{v^2} \frac{d^2 u}{dt^2} \quad \text{--- (7)}$$

where  $\nabla^2$  is the Laplacian operator defined

$$\nabla^2 = \frac{d^2}{dx^2} + \frac{d^2}{dy^2} + \frac{d^2}{dz^2}$$

Eqn (7) represents the diff eqn for a wave propagating in any 3D space

Soln

The solution of 3D wave eqn can be

$$u(x, y, z, t) = a e^{i(\vec{k} \cdot \vec{r} \pm \omega t + \phi)}$$

where  $a$  &  $k$  are constants & they are the amplitude and phase of the wave respectively

$\vec{k} = k_x \hat{i} + k_y \hat{j} + k_z \hat{k}$  is a vector along the direction propagation and is called propagation vector

$$|\vec{k}| = \sqrt{k_x^2 + k_y^2 + k_z^2} \quad \&$$

$$\vec{r} = x \hat{i} + y \hat{j} + z \hat{k}$$

### Transverse Wave in a stretched string

consider a string of length  $l$ , stretched b/w two points A & B by a tension. Let it be plucked at the centre and let free. It vibrates transversely. These vibrations are simple harmonic. Let the normal position of the string correspond to  $x$  axis & the displacement be along  $y$  axis the force acting to bring any element of the string back to equilibrium position is the component of tension acting at right angle to it. Consider a small element of length  $\Delta x$ . The tangents at P & Q make angle  $\theta_1$  &  $\theta_2$  with the horizontal resolving the tension along  $x$  axis &  $y$  axis

net force on  $PQ$  acting on  
 $x$  &  $y$  directions are

$$f_x = T \cos \theta_2 - T \cos \theta_1$$

$$f_y = T \sin \theta_2 - T \sin \theta_1$$

For small oscillations  $\theta_1$  &  $\theta_2$  are small

$$\cos \theta_1 = \cos \theta_2 = 1$$

$$\text{also } \sin \theta_1 = \tan \theta_1 \text{ \& } \sin \theta_2 = \tan \theta_2$$

$$\text{Then } f_x = 0$$

$$f_y = T \tan \theta_2 - T \tan \theta_1$$

so net force acting on element  $\Delta x$  in the displaced position is along  $y$ -axis

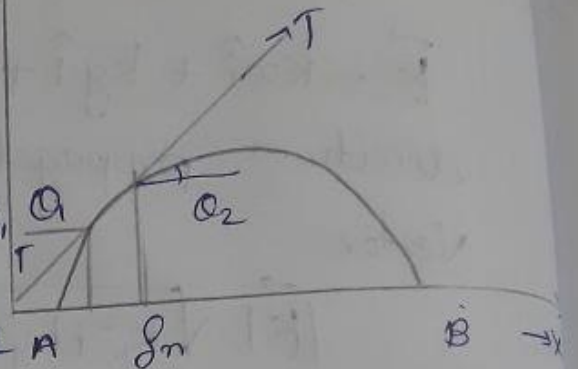
$$f_y = T (\tan \theta_2 - \tan \theta_1)$$

$$T \Delta \tan \theta$$

$$T \Delta \int \frac{dy}{dx}$$

If  $\frac{m}{\Delta x}$  is mass per unit length of string, mass of element  $\Delta x = m \Delta x$

$$\text{acceleration} = \frac{d^2 y}{dx^2}$$





$$m \int \frac{d^2 y}{dt^2} = T \int \frac{dy}{dn}$$

$$m \frac{d^2 y}{dt^2} = T \frac{dy}{dn}$$

$$\frac{m}{T} \frac{d^2 y}{dt^2} = \frac{d^2 y}{dn^2}$$

$$\frac{d^2 y}{dn^2} = \frac{m}{T} \frac{d^2 y}{dt^2}$$

This is the differential eqn of a vibrating string  
comparing this eqn by standard wave eqn

$$\frac{d^2 y}{dn^2} = \frac{1}{v^2} \frac{d^2 y}{dt^2}$$

$$\frac{1}{v^2} = \frac{m}{T}$$

$$v^2 = \frac{T}{m}$$

or  $v = \sqrt{T/m}$  Velocity of transverse wave on a stretched string

$$v = v \lambda$$

$$v = \frac{v}{\lambda} \quad \text{or } v = \frac{1}{\lambda} \sqrt{T/m} \Rightarrow \text{Frequency of}$$

transverse wave developed in a stretched string.

## Module ②

### Interference

The re modification of light energy due to the superposition of the light waves of the same amplitude same frequency and of constant phase difference is called interference

OR

The phenomenon of interference of light is due to the superposition of two or more light waves by the same amplitude, same frequency and of constant phase difference

### Superposition principle

According to superposition principle when two or more waves meet in a region, the resultant displacement in the region is the vector sum of the individual displacements

ie,  $y_1 = a_1 \sin \omega t$  &  $y_2 = a_2 \sin(\omega t + \delta)$

The resultant displacement  $y = y_1 + y_2$

Resultant amplitude

$$A^2 = a_1^2 + a_2^2 + 2a_1a_2 \cos \delta$$

when  $\delta = 0, 2\pi, 4\pi \dots 2n\pi$

$$A^2 = (a_1 + a_2)^2 \Rightarrow A = a_1 + a_2 \Rightarrow \text{Maximum}$$

when  $\delta = \pi, 3\pi, 5\pi \dots (2n+1)\pi$

$$A^2 = (a_1 - a_2)^2 \Rightarrow A = a_1 - a_2 \Rightarrow \text{Minimum}$$

Condition for constructive interference (For maxima)

$\Rightarrow$  when crest of one wave meets with crest of another  
~~trough~~ trough of one meets with trough of other then

the resultant amplitude ~~and~~

to maximum  $\Rightarrow$  constructive interference

Condition  $\Rightarrow$

$$\text{Phase difference} = 2n\pi, n = 0, 1, 2, \dots$$

$$\text{path difference} = n\lambda, n = 0, 1, 2, \dots$$

$$\lambda \text{ path difference} = 2\pi$$

## Condition for destructive interference (for minima)

when crest of one wave meets with trough of another, then, the resultant intensity and amplitude is minimum  $\rightarrow$  Destructive interference

Condition  $\vdash$  phase difference  $(2n+1)\pi$ ,  $n = 0, 1, 2, \dots$

path difference  $\rightarrow (2n+1)\frac{\lambda}{2}$ ,  $n = 0, 1, 2, \dots$

## Condition for permanent interference pattern

- $\Rightarrow$  Source must be coherent
- $\Rightarrow$  Light waves from one source should superimpose at the same time and at the same place
- $\Rightarrow$  Two sources should be very close to each other

## Coherence

The source of light is said to be coherent, when the light waves emerging from the source must have same amplitude, same frequency and constant phase difference

Eg: Two slits illuminated by a monochromatic screen

- $\Rightarrow$  A source of light and its reflected light image
- $\Rightarrow$  Two refracted images of same source

## Two Types of Interference

Interference is divided into two types depending on the mode of production of interference pattern

### ① Interference produced by the division of wavefront

The incident wavefront is divided into two points by ~~reflection~~ reflection, refraction, diffraction and total internal reflection. Now these two divided points of ~~turns~~ turn unequal distance through the medium and then they combine together to produce interference pattern.

Eg: Young's double slit experiment.

### Interference produced by the division of Amplitude

The amplitude or intensity of the incident light is divided into two parts by parallel reflection or refraction. These two divided parts of wavefront travel unequal distances through the medium and then they combine together to produce interference pattern.

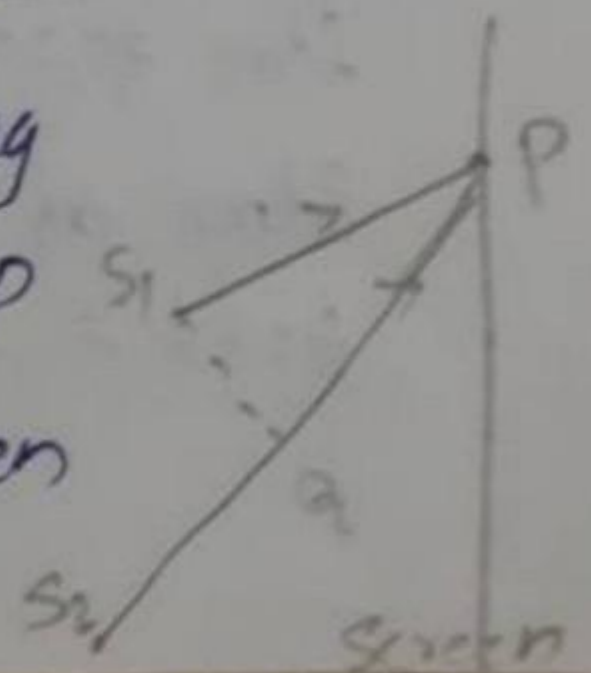
Eg: Newton's experiment

conditions for constructive & destructive interference

$S_1$  &  $S_2$  two coherent sources emitting

waves of wavelength  $\lambda$ . Consider a point P

on a screen the path difference between the point P is  $S_2P - S_1P = S_2Q$



For constructive interference at  $P$ , ~~to~~ to produce a bright point at  $P$ , the path difference between the waves meeting  $P$  must be ~~an~~ <sup>even</sup> an integral multiple of wavelength  $\lambda$

$$\text{i.e., } S_2Q = 0, \lambda, 2\lambda, \dots$$

$$\text{or } \boxed{S_2Q = n\lambda}$$

$\Rightarrow$  For destructive interference at  $P$ , the path difference between the waves ~~are~~ meeting  $P$  must be an odd multiple of  $\lambda/2$

$$\text{i.e., } S_2Q = \lambda/2, 3\lambda/2, 5\lambda/2, \dots$$

$$= (n + 1/2)\lambda$$
$$\boxed{S_2Q = (2n + 1) \frac{\lambda}{2}} \quad n = 0, 1, 2, 3, \dots$$

Interference of light ~~produced~~ <sup>reflected</sup> from plane parallel thin film

When a beam of light falls on a thin transparent film, a part of light is reflected from one top surface of the film and a part of light is reflected from the lower surface of the film. These two reflected rays interfere. If the incident light is white, the film appears beautifully coloured. This is why a film of oil on the surface of water or a soap bubble appears coloured in sunlight.

## Diffraction

It is the phenomenon of bending of light round the edges of an obstacle or encroachment of light into the geometrical shadow of the obstacle

### Fresnel diffraction

Statement: The diffraction pattern created by the waves which is passing through an aperture or around an object, when viewed from relatively close to the object

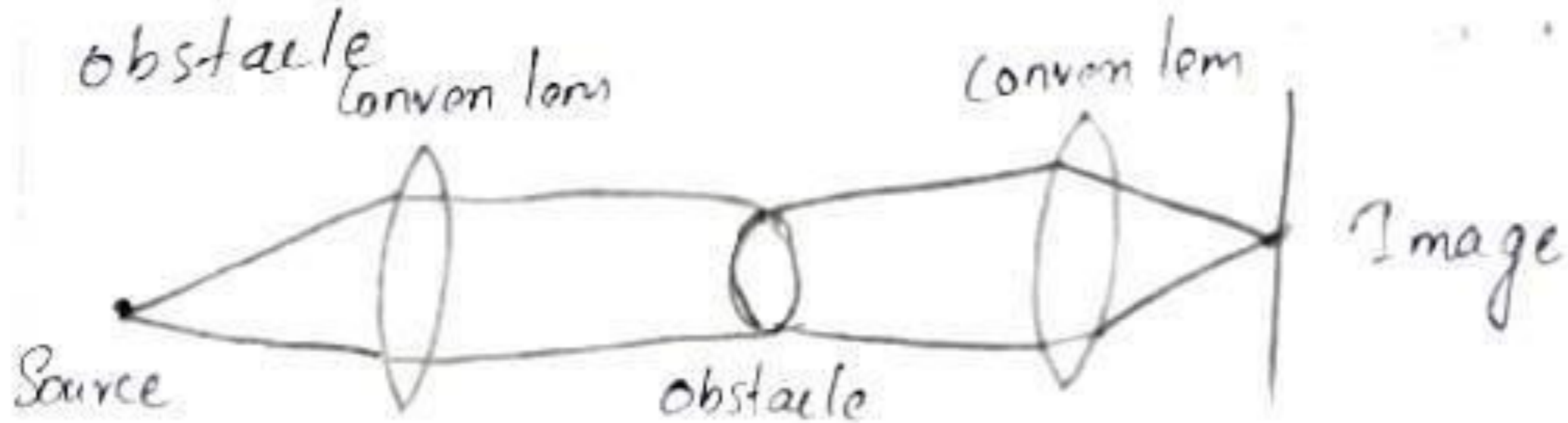
OR

The diffraction of light, when the source (light) and screen are at finite distance from the obstacle

- The wavefront falling on the obstacle is spherical or cylindrical
- Lens are not used

### Fraunhofer diffraction

The diffraction causes due to a source of light which is at infinite distance from the obstacle



- The wave front falling on the obstacle are plane
- Convex lens are used (converging lens)

## Fraunhofer diffraction at a single slit

A plane wave front of monochromatic light of wavelength ( $\lambda$ ) passes through the slit AB with width  $a$ . Huygens principle states that each point on the wavefront behaves like a secondary wave. So slit AB is an source of point. The centre of the known as 'O'.

The waves proceeding from sources are straight and parallel to the DP <sup>focused on</sup> ~~covering~~ the point 'P'. They rays are covering equal path and same phase without any path difference and resolves the point P and these leads to maximum brightness due to constructive reinforcement of waves. Thus Bright band is occurred at the point P, known as zero order central maximum.





A point on the screen which is just above the point with an angle  $\theta$ , a line  $AM$  is drawn ~~per~~  $\perp$  to  $BM$  and beyond this point the waves have same paths

$BM$  is the path difference between two

3/10

So  $BM = a \sin \theta$  — (1)

(consider triangle  $ABM$   $\sin \theta = \frac{BM}{AB}$ )

$BM = \lambda$  (wave length of light)

(1)  $\rightarrow \lambda = a \sin \theta$  — (2)

The total distance between the slits is  $a$ . so by considering the midpoint  $O$   $AO$  and  $BO$  is  $a/2$  where  $O$  is half of  $a$  (total distance)

$\therefore AO = BO = a/2$  — (3)

The waves proceeding from  $O$  and  $B$  are traveling along  $OM$  and  $BM$  reaches the point the

point (due to lens)

From the equation. no ②

$$n\lambda = (a+b) \sin \theta$$

∴  $n=1 \rightarrow$  First order principle maxima

$n=2 \rightarrow$  Second order principle maxima

$n=3 \rightarrow$  Third order principle maxima

There are  $N$  lines / unit length of grating ~~there are~~

There  $N$  sub are.

$$N(a+b) = 1 \rightarrow \text{unit length}$$

$$a+b = \frac{1}{N} \quad \text{--- ③}$$

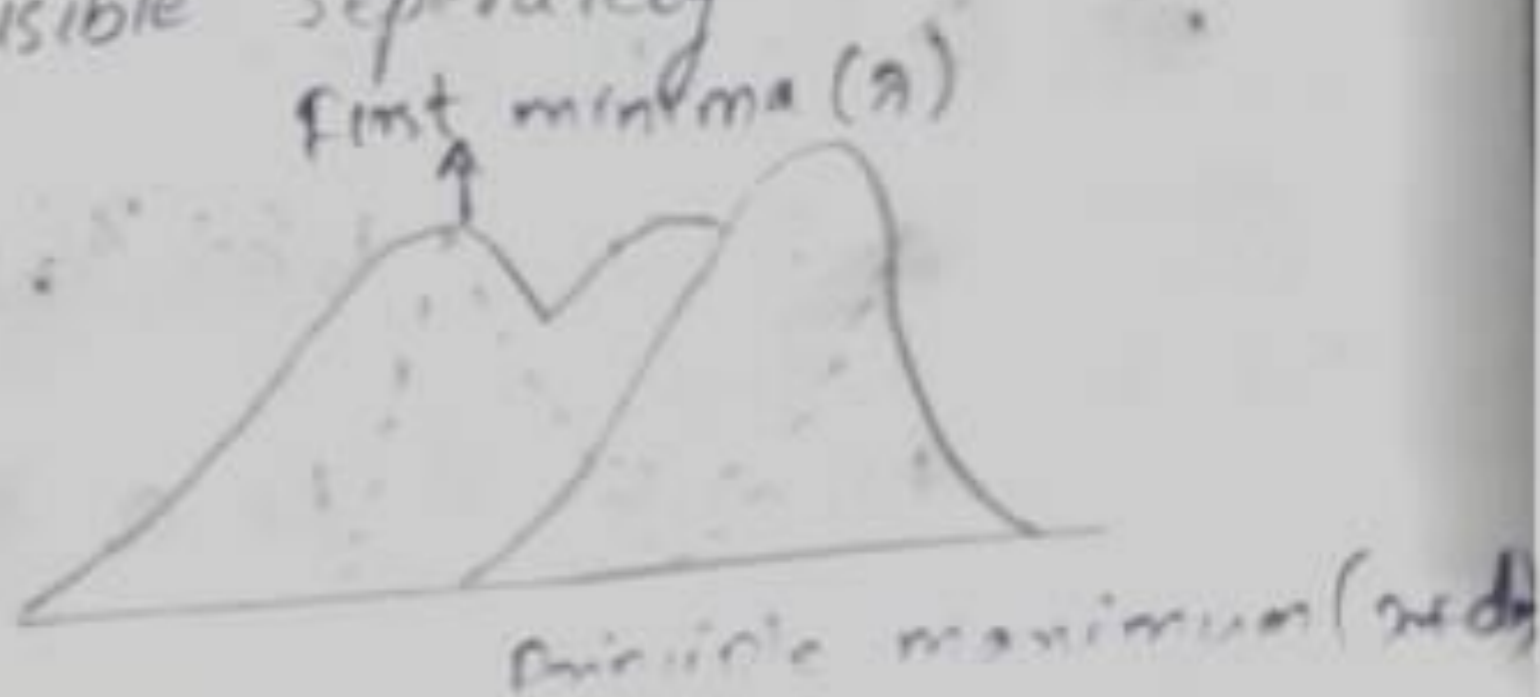
Sub eqn ③ ① ②  $\rightarrow$

$$\frac{1}{N} \sin \theta = n\lambda$$

$$\text{or } \boxed{\sin \theta = nN\lambda} \rightarrow \text{Grating law or Equation}$$

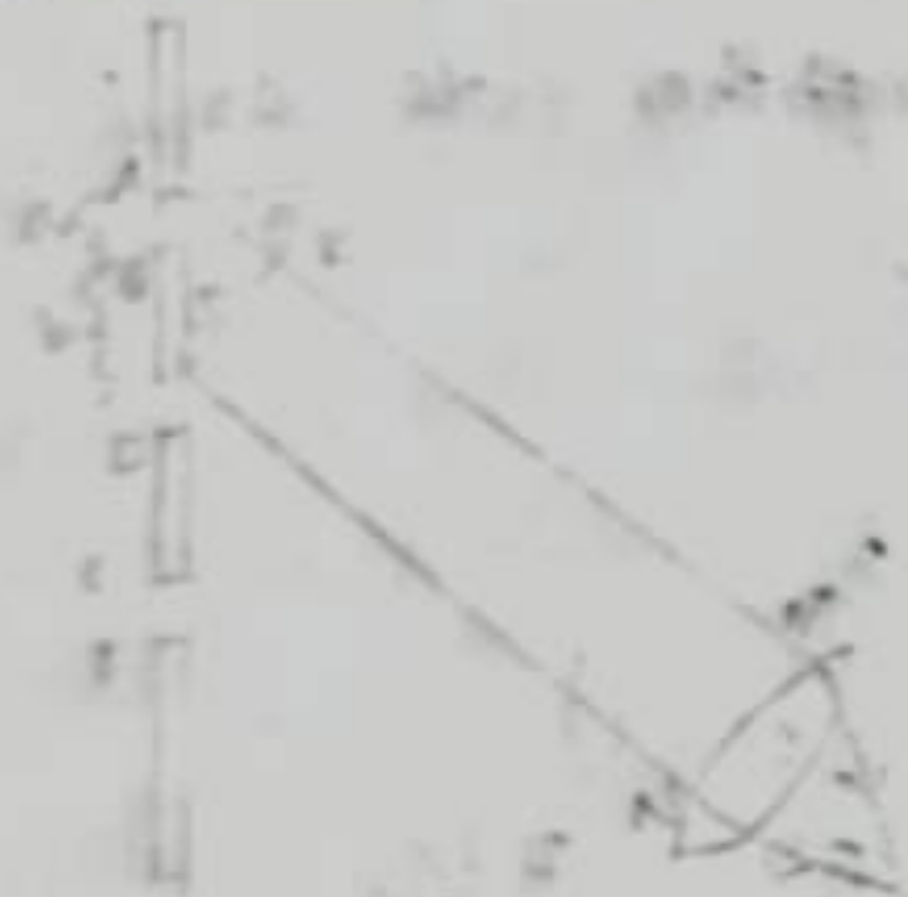
Rayleigh's Criterion For Resolution of Spectral lines

It states that when one principle maximum falls on the other first minimum, ~~the~~ some order then both the waves will be visible separately



## Diffraction Grating by slit.

Two waves from the corresponding points A & C of adjacent slits. Let  $\lambda$  be the wavelength and  $\theta$  be the angle of diffraction with the normal to the grating. They travel along Am and CN & AK perpendicular to the line Am path. Path difference is AK.



$\triangle ACK$

$$\sin \theta = \frac{AK}{AC}$$

$$\therefore AK = AC \sin \theta$$

$$AK = a + b \sin \theta \quad [AC = a + b]$$

Where AK is the path difference (represented by  $n\lambda$ )

$$\therefore n\lambda = (a + b) \sin \theta \quad \text{--- (2) (when two waves interfere constructively)}$$

The waves of wavelength  $\lambda$  originate

from different corresponding points with diffracted angle  $\theta$  reinforce and give a bright line at

## Resolving Power Of Grating

Resolving power of grating is defined as the measure of its ability to spatially separate two wavelengths.

" In Grating there are  $N$  slits and path difference when they reach a point on the screen. the path difference between the waves from adjacent slits is changed by  $\lambda/N$ . If grating has two halves then the path difference is  $\lambda/2$ .

According to Rayleigh's criterion for Resolution two separate lines are just resolved when the principle maximum of  $n$ th order for  $\lambda + d\lambda$  falls at the first maximum of the same order for  $\lambda$ . Then the angle difference is same.

$n$ th Order principle maximum for  $\lambda + d\lambda$  is

$$(a+b) \sin \theta = n(\lambda + d\lambda) \quad \text{--- (1)}$$

$(a+b)$  = grating constant  
 $n$ th Order maxima

$$a+b \sin \theta = n\lambda + \lambda/N_1 \quad \text{--- (2)}$$

$N_1 \rightarrow$  total no of slits

Substitute ② in ①

$$n(\lambda + d\theta) = n\lambda + \lambda/N,$$

$$n\lambda + n d\theta = n\lambda + \lambda/N, \quad \text{--- ③}$$

By simplifying above equation

$$n\lambda + nd\theta = n\lambda + \lambda/N,$$

$$N d\theta = \lambda$$

$$N_1 N = \lambda d\theta$$

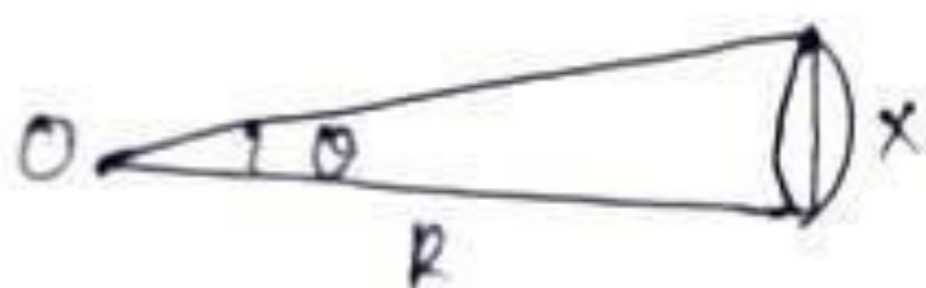
→ Resolving power of grating

When we use lens the above eqn can be written as

$$\theta_N = 1.22 \lambda/D$$

The condition for Rayleigh's criterion for minimum angle of resolution using a lens with diameter 'D' at a wave length  $\lambda$  is given by

$$\theta_{\text{min}} = \frac{1.22 \lambda}{N}$$



Dispersive power of a grating

It is known as the ratio of change in angle of diffraction to the corresponding change in wavelength

let  $\lambda$  and  $\lambda + d\lambda$  with angles  $\theta$  and  $\theta + d\theta$   
The dispersive power of grating is  $\frac{d\theta}{d\lambda}$   
 $\therefore n^{\text{th}}$  maxima for a wave length  $\lambda$

$$(a+b) \sin \theta = n\lambda \quad \text{--- (1)}$$

differentiating both sides

$$(a+b) \cos \theta d\theta = n d\lambda$$

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b) \cos \theta}$$

$$\frac{d}{d\lambda} = \frac{n}{(a+b) \cos \theta}$$

so  $\boxed{\frac{d\theta}{d\lambda} = \frac{n}{(a+b) \cos \theta}}$

dispersive power  
Formula

# NANOSCIENCE

Nanoscience is the study of and application of structure and materials that have dimensions at the nanoscience level. Nanoscience is the study of nanomaterials and their properties, and the understanding of how these materials, at the molecular level, provide novel properties and physical, chemical and biological phenomena that have been successfully used in innovative way in a range of industries.

Feynman's 1959 talk is often cited as a source of inspiration for Nanoscience but it was only published as a scientific paper in 1992.

## Nanotechnology

Nanoscience is the science and technology of object at the nanoscale level, the properties of which differ significantly from that of their constituent material at the macroscopic or even microscopic scale. It is a multidisciplinary field that encompasses understanding and control of matter at about 1-100nm, leading to development of innovative and revolutionary applications.

## Difference b/w Nanotechnology & Nanoscience

Nanoscience and Nanotechnology are the study & application of extremely small things, The materials with nanometre dimensions. Nanoscience is where atomic physics converges with the physics & chemistry of complex systems. Nanoscience technology is the science and technology of objects at the nanoscale level, the properties of which differ significantly from that of their constituent material at the macroscopic or even microscopic scale. When we're talking about a scale an order of magnitude of size, or length. Nanoscience is the study of structures and materials on the nanoscale.

Nanotechnology is a multidisciplinary field that encompasses understanding and control of matter at about 1-100nm, leading to development of innovative and revolutionary applications. It encompasses nanoscale sciences, engineering and technology in addition to modeling and manipulation of matter on an atomic, molecular & supermolecular scale. Nanoscience is about the phenomena that occurs in systems with nanometre dimensions.

& it involves understanding the fundamental interactions of physical systems confined to nanoscale dimensions and their properties.

## INCREASE IN SURFACE AREA TO VOLUME RATIO

When size of the particle ↓ the ratio of surface area to volume ↓

"The ratio of surface area to volume (SAVR) plays a vital role in nanoscience and nanotechnology. The ratio is the amount of surface area per unit volume of an object."

Cube :-

consider a cube with a side length of 10, volume of the cube is  $10^3 = 10 \times 10 \times 10$  or  $a^3 \Rightarrow$

where 'a' is the side of a cube:

area is  $10 \times 10 = 100$  ( $a^2$ ), cube has 6 sides.

$$\begin{aligned} \text{Total surface area} &= 6(10 \times 10) \\ &= 6 \times 10^2 \\ &= 600. \end{aligned}$$

Surface volume ratio :-

$$\frac{\text{Surface area}}{\text{Volume}} = \frac{600}{10^3} = 0.6$$

$\Rightarrow$  when the same cube with side 'a' is 5

$$\text{Volume is } a^3 = 5 \times 5 \times 5 = 125$$

$$\text{area} = 5 \times 5 = 25$$

$$\text{cube has 6 faces} = 6 \times a^2 = 6 \times 25 = 150$$

So SAVR (Surface area to the volume ratio) is,

$$\frac{\text{Surface area}}{\text{Volume}} = \frac{150}{125} = 1.2$$

So it is proved that when size ↓ the surface area to the vol. ratio ↑. So it is proved that nanomaterials has more (enhanced) 'SAVR' than the micro or macro molecules.

$\Rightarrow$  Solve

If side of a cube has length of 1

$$\text{Volume} = 1^3 = 1$$

$$\text{area} = 1^2$$

$$\frac{\text{area}}{\text{Vol}} = \frac{1}{1} = 1$$

$\Rightarrow$  Solve

Derive an eqn when side of cube is 's'

If 's' is the side

$$\text{Volume of the cube} = s^3$$

$$\text{Surface area of cube} = 6s^2$$

$$\text{Ratio of surface area to volume} = \frac{6s^2}{s^3}$$

$$= \frac{6}{s}$$

## Quantum Confinement

The change in electronic and optical properties of the material when its size is reduced (10nm or less than 10nm) is considered as Quantum Confinement.

## Quantum Confinement in One Dimension

### Quantum Confinement

The optical property & electrical property changes when the material sample is of sufficiently small size (10nm or less than 10nm)

When the length of a semiconductor is reduced to the same order of the exciton radius to a few nanometers, quantum mechanical confinement effect occurs & the exciton properties are modified. These types of quantum confinement structures are quantum well (QW), Quantum wire (QR) & Quantum dot (QD)

### Exciton

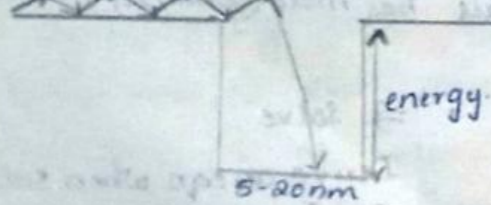
It is a bound state of an  $e^-$  & hole can empty electron state in a valence band, which is free to move through a nanometallic crystal.

### Exciton radius

It is the distance b/w  $e^-$ -hole pair.

### Quantum Well

It is a well thin layer which can confine particles (quasi) ( $e^-$  or holes) in the dimension  $\perp$  to the layer surface, whereas the movement of particles in other dimension is not restricted. The thickness of one quantum well is  $\approx$  5-20 nm.



### One dimensional Quantum Confinement

The optical & electrical property of the material sample changes & forms in a one dimensional (1D) such materials are known as 1D quantum confinement.

Electrons confined in one dimensional (1D) quantum well (thin film)  $e^-$  can easily move in 2-D, so one dimensional is quantified.

$e^-$  confined in 2-D quantum wires,  $e^-$  can easily move in 1-D, so 2-D is confined.

$e^-$  confined in 3-D, quantum dots (QD) so 3-D is quantized.

### Nanosheet

A 2-D nanostructure with thickness (1 to 100nm)

eg: Graphene

Example:- ① Silicon nanosheets:- are being used to prototype future generation transistors (5nm)

② Carbon nanosheets:- A graphene alternate, used as electrodes in super capacitors.

### Nanowire

A nanostructure with the diameter of the order of nanometers ( $10^9$ nm) ratio of length to width is greater than 1000 it's mainly used for transistors (MOSFET)

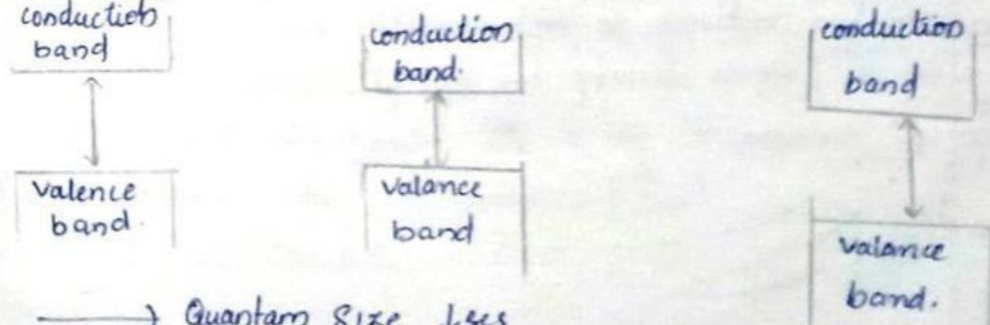
### Quantum Wire

A formation of confined 2-Dimension system by making a thin wire of the selected semiconductor.

Trapped excitons in 2-D quantum confinement but it is free to move in 1-D only along the wire.

### Quantum Dots:-

A semiconductor nanocrystalline materials with diameter usually at 2-10nm, it can produce distinctive colours determined by the size and shape.



Due to quantum confinement the electrons come back to the valence band & uses  $\Delta E$  energy (light)

According to quantum mechanics energy of photons relates to the wavelength (colour) of photons. This means emitted colour



depends on band gap. Various size of quantum dots results in different colouring. Small size emits blue colour light but larger band gap where as bigger size will emit red colour light with small band gap (TV screens - LED TVs)

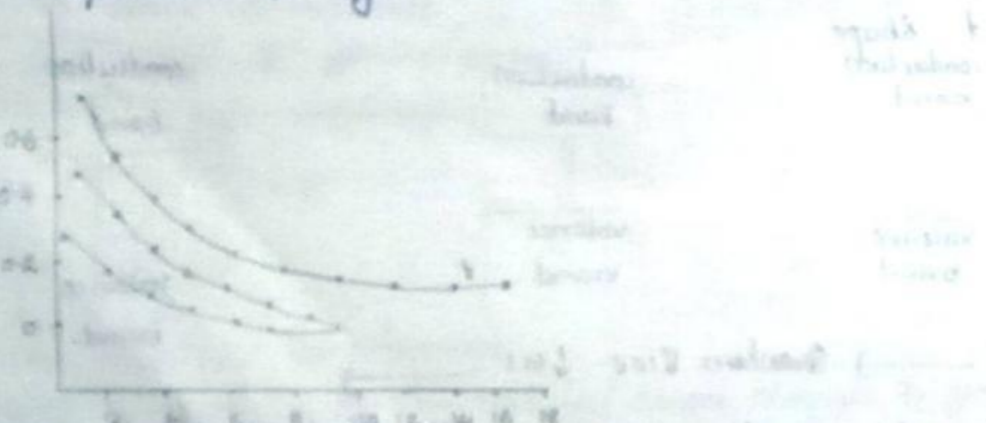
### Optical Properties

Nanocrystalline systems have interesting optical properties. Depending on the particles size, same substance shows different colours. Gold nanospheres of 100nm appears orange to colour while that of 50nm size appears green. In the case of nanosized semiconductor particles quantum effects come into play and optical properties can be varied merely by controlling its size. These particles can be made to emit or absorb specific wavelength of light by varying its size. The linear and non-linear properties of such materials can be tuned in the same way. Nanomaterials such as tungstic oxide gel is explored for large electronic display devices.

### Magnetic properties

The strength of a magnet is measured in terms of coercivity and saturation magnetization values. These values increase with a decrease in grain size and with increase in specific surface area (surface area per unit volume). Therefore nanomaterials present good properties in this field.

In small particles a large fraction of the atoms reside at the surface. These atoms have lower co-ordination numbers than the interior atoms. The magnetic moment is determined by the local co-ordination number. Fig 4 shows the calculated dependence of magnetic moment on the nearest coordination number.



It is clear that as the co-ordination number decreases the magnetic moment increases. In short, small particles are more magnetic than bulk materials. Even nanoparticles of nonmagnetic solids are found to be magnetic i.e., at small sizes, the clusters become spontaneously magnetic.

### Mechanical properties

Most metals are made up of small crystalline grains. The boundaries btw the grains slow down or arrest the propagation of defects when the mechanical material is stressed, thus giving its strength. If the grains are nanoscale in size the interface area is greatly increasing, which increases its strength. For eg: nanocrystalline (substance) nickel is as strong as hardened steel. Because of the nanosize, many of their mechanical properties such as hardness, elastic modulus, fracture toughness, scratch resistance and fatigue strength are modified.

Some observations on the mechanical behaviour of nanostructured materials are:

- 1) 30-50% lower elastic modulus than conventional materials.
- 2) 2-7 times higher hardness.
- 3) Super plastic behaviour in brittle ceramics.

The experimental behaviour of hardness measurements show different behaviour namely positive slope, zero slope, and negative slope depending on the grain size, when it is less than 20nm. Thus the hardness, strength and deformation behaviour of nanocrystalline materials is unique and not well understood.

Super plasticity is another phenomenon that has been found to occur in nanocrystalline materials at some what lower temperature and higher strain rates.

### Heisenberg's Uncertainty Principle

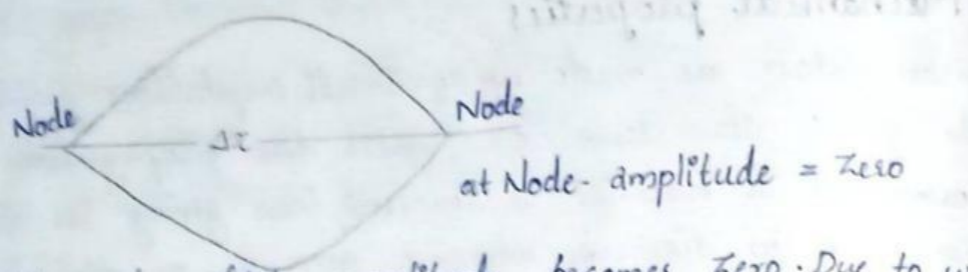
It states that it is impossible to determine position (x) and the momentum (p) of a particle with absolute precision.

#### Statement

In any simultaneous determination of position and momentum of the particle, the product of uncertainty  $\Delta x$  (or possible error) in the x-co-ordinate of a particle in motion and uncertainty  $\Delta p_x$  in the x-component of momentum is of the order of or greater than  $h = (1.054 \times 10^{-34} \text{ Js})$

$$\Delta x \Delta p_x \geq h$$

→ Proof  
Consider a particle (wavepacket) moving in x-axis. The envelope of the wave packet moves with a velocity equal to particle velocity. When the wave packet extends  $\Delta x$  (finite distance), the two points at which the amplitude of the wave packet becomes zero and it will be repeated successively.



Nodes means the points at which amplitude becomes zero. Due to wave nature of the particle position of the particle will have minimum error equal to distance  $(\Delta x)$

The amplitude of the wave packet is,

$$R = 2A \cos \left[ \frac{\Delta \omega}{2} t - \frac{\Delta k}{2} x \right] \quad (1)$$

At node amplitude is zero.

$$\text{So, } 0 = 2A \cos \left[ \frac{\Delta \omega}{2} t - \frac{\Delta k}{2} x \right] \quad (2)$$

Since  $2A \neq 0$  (taking  $2A$  to LHS)

$$\cos \left[ \frac{\Delta \omega}{2} t - \frac{\Delta k}{2} x \right] = 0 \quad (3)$$

$$\left[ \frac{\Delta \omega}{2} t - \frac{\Delta k}{2} x \right] = 0$$

When  $\cos$  is  $(2n+1) \frac{\pi}{2}$  i.e.,  $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \dots$  (4)

we know there are two nodes so the positions are also two.

$\therefore$  positions of two nodes are two i.e.,

$$\text{1st position } \frac{\Delta \omega}{2} t - \frac{\Delta k}{2} x_1 = (2n+1) \frac{\pi}{2} \quad (5)$$

$$\text{2nd position } \frac{\Delta \omega}{2} t - \frac{\Delta k}{2} x_2 = (2n+1) \frac{\pi}{2} + \pi = (2n+3) \frac{\pi}{2} \quad (6)$$

on simplifying (5) and (6) (subtraction)

$$\frac{\Delta k}{2} (x_2 - x_1) = \pi \quad (7)$$

$$\Delta k (x_2 - x_1) = 2\pi$$

$$x_2 - x_1 = \frac{2\pi}{\Delta k} \quad (8)$$

$$\Delta x = \frac{2\pi}{\Delta k} \quad (9)$$

This is the fundamental error in the measurement of the position of the particles.

$$k = \frac{2\pi}{\lambda} \quad (10)$$

$$\lambda = h/p_x \quad (11)$$

where  $h \rightarrow$  Planck's constant

$\Delta p_x \rightarrow$  momentum of the particle in x-axis.

$$\text{Since, } k = \frac{2\pi}{\lambda}$$

Sub eqn (11) in eqn (10)

$$k = \frac{2\pi}{h/p_x} ; k = \frac{2\pi p_x}{h}$$

$$\text{i.e., } \Delta k = \frac{2\pi (\Delta p_x)}{h} \quad (12)$$

from eqn no 9  $\Delta x = \frac{2\pi}{\Delta k}$

Sub eqn (12) in eqn (9)

$$\Delta x = \frac{2\pi}{\frac{2\pi \Delta p_x}{h}} = \frac{h}{\Delta p_x}$$

$$\Delta x = \frac{h}{\Delta p_x}$$

According to superposition of waves.

$$\Delta x = \frac{1}{\Delta k} \quad \text{or} \quad \Delta k = \frac{1}{\Delta x}$$

$$\frac{1}{\Delta x} = \frac{2\pi \Delta p_x}{h} \quad (\text{from eqn 12})$$

$$\frac{1}{\Delta x \Delta p_x} = \frac{2\pi}{h}$$

$$\Delta x \Delta p_x = \frac{h}{2\pi}$$

$$\Delta x \Delta p_x = \hbar$$

$$\frac{h}{2\pi} = \hbar$$

Thus,  $\Delta x \Delta p_x \geq \hbar$

Que A microscope using photons is employed to locate an  $e^-$  in an atom  $0.2 \text{ \AA}$ . what is the uncertainty in the momentum of the  $e^-$  located in this solution. Given  $\Delta x = 0.2 \text{ \AA} = 2 \times 10^{-11} \text{ m}$   $\Delta P = ?$

ans. Since we know that the Uncertainty principle

$$\Delta x \Delta P_x = \frac{h}{2\pi}$$

$$\Delta P_x = \frac{h}{2\pi \Delta x}$$

$$\Delta P_x = \frac{6.626 \times 10^{-34}}{2\pi \times 2 \times 10^{-11} \text{ m}}$$

$$= 5.27 \times 10^{-24} \text{ kg m/s}$$

Que Show that the uncertainty in the location of the particle is equal to the de Broglie wavelength the uncertainty in its velocity is equal to its velocity.

ans. Solution, Given  $\Delta x = \lambda$

Since we know that by uncertainty principle

$$\Delta x \Delta P_x = \frac{h}{2\pi \Delta x}$$

$$\Delta P_x = \frac{h}{2\pi \Delta x}$$

$$\Delta x \Delta P_x = \hbar$$

$$\lambda \Delta P = \hbar$$

$$\Delta P = \frac{\hbar}{\lambda}$$

$$\text{we have } \frac{\hbar}{\lambda} = p \quad \text{ie } \Delta P = P$$

$$\Delta P = P$$

$$m \Delta V_x = m V_x$$

$$\frac{m}{m} \Delta V_x = V_x$$

$$\Delta V_x = V_x$$

Que A microscope along using photons is employed to located an  $e^-$  in an atom  $5 \text{ \AA}$ . what is Uncertainty in the momentum of the  $e^-$  located in this.

ans.  $\Delta x = 5 \text{ \AA} = 5 \times 10^{-12} \text{ m}$   $\Delta P = ?$

$$\Delta x \Delta P_x = \frac{h}{2\pi}$$

$$\Delta P_x = \frac{h}{\Delta x 2\pi}$$

$$= \frac{6.626 \times 10^{-34}}{5 \times 10^{-12} \times 2\pi}$$

$$= 2.1091 \times 10^{-23} \text{ kg m/s}$$

### Application of Heisenberg's Uncertainty principle

(\*) Absence of free electrons.

$$E^2 = P^2 c^2 + m_0^2 c^4$$

$$\Delta x \Delta P_x = \frac{h}{2\pi}$$

$$\Delta P_x = \frac{h}{2\pi \Delta x} = \frac{6.626 \times 10^{-24}}{2\pi \times 1 \times 10^{-14}}$$

$$\Delta P_x = 1.054 \times 10^{-20} \text{ kg m s}^{-1}$$

$$E^2 = (1.054 \times 10^{-20} \times 3 \times 10^8)^2 + (9.1 \times 10^{-31})^2 (3 \times 10^8)^4$$

$$= 9.929 \times 10^{-24} \text{ joule}$$

$$= P$$

→ Absence of free electrons

According to theory of relativity energy of a particle is given by the relation

$$E^2 = P^2 c^2 + m_0^2 c^4$$

where  $P$  = momentum

$m_0$  = rest mass of the particle

According to Heisenberg's principle

$$\Delta x \Delta p_x = \frac{h}{2\pi}$$

The diameter of the nucleus is  $10^{-14}$  m, so the maximum possibility of the particles is within its diameter thus the position of the particle is in  $10^{-14}$  m.

$$\therefore \Delta x = 10^{-14} \text{ m}$$

$$\Delta x \Delta p_x = \frac{h}{2\pi}$$

$$\Delta p_x = \frac{h}{2\pi \Delta x} = \frac{6.63 \times 10^{-34}}{2 \times 3.14 \times 10^{-14}} = 1.05 \times 10^{-20} \text{ kg m/s}$$

For electron of minimum momentum, the minimum energy is given by

$$E_{\text{min}} = p_{\text{min}}^2 c^2 + m_0^2 c^4$$

$$= (1.055 \times 10^{-20} \times 3 \times 10^8)^2 + 9.1 \times 10^{-31} \times (3 \times 10^8)^4$$

$$= 3 \times 10^8 \sqrt{1.113 \times 10^{-40}}$$

$$= 3.1648 \times 10^{-12} \text{ J}$$

Converting into eV

$$\therefore E_{\text{min}} = \frac{3.1648 \times 10^{-12}}{1.6 \times 10^{-19}} \text{ eV} \approx 20 \text{ keV}$$

If free  $e^-$  exists the nucleus must have minimum energy about 20 keV. But the minimum required K.E which a  $\beta$ -particle, emitted from radioactive nucleus is at 4 keV.

### Quantum Mechanics

classical physics couldn't properly explain many physical phenomena, because it deals with microscope particles. Max plank in 1900, put forward the quantum theory to explain black body radiation.

Einstein introduced the idea of light quantum or photon. Particle nature of radiation was stressed in these theories.

But wave nature of radiation was essential to explain interference, diffraction etc. In 1924, Louis de Broglie suggested wave particle duality. In 1926, Schrodinger developed the wave particle mechanics. PAM Dirac unified wave mechanics and matrix mechanics to setup a general formulation called Quantum

mechanics. It deals with microscopic particles.

### WAVE NATURE OF PARTICLES

In 1924, De-broglie predicted that like radiation, particle has a dual nature, particle and wave nature.

de-broglie hypothesis.

All moving particle is associated with a couple called matter wave or de-broglie wave and its wavelength is known as de-broglie wavelength which is given by,

$$\lambda = \frac{h}{p} \quad (1)$$

where  $h \rightarrow$  Planck's constant  
 $6.626 \times 10^{-34} \text{ J s}$

$p \rightarrow$  momentum of the particle  
 $= mv$

According to mass-energy relation

$$E = mc^2 \quad (1) \quad \text{particle nature}$$

we know the relation (wave-nature)

$$E = h\nu \quad (2)$$

equating (1) and (2)

$$mc^2 = h\nu$$

$$mc = \frac{h\nu}{c}$$

$$p = \frac{h\nu}{c}$$

substitute  $\frac{\nu}{c} = \frac{1}{\lambda}$

$$p = \frac{h}{\lambda}$$

$$\lambda = \frac{h}{p}$$

but  $mc = p$

we have,

$$v = \nu \lambda \quad \text{or} \quad c = \nu \lambda \quad p = \frac{h}{\lambda}$$

$$\therefore \frac{\nu}{c} = \frac{1}{\lambda}$$

$$\lambda = \frac{h}{p}$$

Que. calculate the wavelength of an electron accelerated by a potential difference of  $V$  volt.

ans Energy of electron

$$E = eV \quad (1)$$

where  $e$  = charge of  $e^-$   
 $= 1.6 \times 10^{-19} \text{ J}$

$V$  = applied potential difference

we have  $KE = \frac{1}{2} m v^2 \quad (2)$

$$\frac{1}{2} m v^2 = eV \quad (3)$$

$$v^2 = \frac{2eV}{m}$$

$$v = \sqrt{\frac{2eV}{m}}$$

$h \rightarrow$  Planck's constant  
 $m \rightarrow$  mass of  $e^- = 9.1 \times 10^{-31} \text{ kg}$   
 $e \rightarrow$  charge of  $e^- = 1.6 \times 10^{-19} \text{ C}$   
 $V \rightarrow$  potential diff in volt.

Then momentum  $P = mv$

$$P = m \sqrt{\frac{2eV}{m}} = \sqrt{\frac{m^2 \cdot 2eV}{m}} = \sqrt{2eVm}$$

$$\text{then } \lambda = \frac{h}{P} = \frac{h}{\sqrt{2meV}} \quad (4)$$

Que. Calculate the wavelength associated with an  $e^-$  under a potential difference of 100V.

ans. For an  $e^-$ ,  $\lambda = \frac{h}{\sqrt{2meV}}$

$$h = 6.625 \times 10^{-34} \text{ Js}$$

$$m = 9.1 \times 10^{-31} \text{ kg}$$

$$V = 100 \text{ V}$$

$$\text{then } \lambda = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 100}} = 1.2 \times 10^{-10} \text{ m}$$

Que. Estimate the de Broglie wavelength of an  $e^-$  moving with a K.E of 100 eV.

ans. we have, for an electron

$$KE = eV$$

$$\frac{1}{2} m v^2 = eV = 100 \text{ eV}$$

$$= 100 \times 1.6 \times 10^{-19} \text{ J}$$

$$\text{then } \lambda = \frac{h}{\sqrt{2meV}}$$

$$\lambda = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 100}}$$

$$\lambda = 1.2 \times 10^{-10} \text{ m}$$

(\* Calculate the de Broglie wavelength of whose KE is 10 keV.

ans.  $KE = eV = 10 \text{ keV}$   
 $= 10 \times 10^3 \times 1.6 \times 10^{-19} \text{ J}$

$$\text{then } \lambda = \frac{h}{\sqrt{2meV}}$$

$$\lambda = \frac{6.625 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 10 \times 10^3 \times 1.6 \times 10^{-19}}}$$

$$\lambda = 1.22 \times 10^{-11} \text{ m}$$

UNCERTAINTY PRINCIPLE (Heisenberg's Uncertainty principle)

It is impossible to have an accurate measurement of two conjugate variables simultaneously. i.e.,

it is impossible to know both the exact position and exact momentum of an object at the same time.

Let the Uncertainty in position =  $\Delta x$

Uncertainty in momentum =  $\Delta p_x$

Then according to Heisenberg's Uncertainty principle

$$\Delta x \Delta p_x \geq \frac{h}{2} \quad \text{where } \hbar = \frac{h}{2\pi}$$

$$\text{or } \Delta x \Delta p_x \geq \hbar$$

Similarly Uncertainty in energy =  $\Delta E$

Uncertainty in time =  $\Delta t$

then  $\Delta E \Delta t \geq \frac{\hbar}{2}$  or

$$\boxed{\Delta E \Delta t \geq \hbar}$$

Application of Uncertainty principle

Q. Absence of electron inside the nucleus

Let the nucleus of the order of  $10^{-14}$  m.

ie,  $\Delta x \approx 10^{-14}$  m

By uncertainty principle

$$\Delta x \cdot \Delta p_x \geq \hbar$$

$$\Delta x \cdot \Delta p_x = \hbar = \frac{h}{2\pi}$$

$$\text{then, } \Delta p_x = \frac{h}{2\pi \Delta x} = \frac{6.625 \times 10^{-34}}{2\pi \times 10^{-14}}$$

$$\Delta p_x = \underline{1.10 \times 10^{-20} \text{ kg m/s}}$$

This momentum contributes to the necessary energy of the nucleus ie,

$$\text{energy of the nucleus} = 1.10 \times 10^{-20} \text{ J}$$

$$\text{energy of } e^- \approx 20 \text{ meV}$$

$$\approx 20 \times 10^6 \times 1.6 \times 10^{-19} \text{ J}$$

$$\approx \underline{8.2 \times 10^{-12} \text{ J}}$$

→ energy of nucleus  $\neq$  energy of  $e^-$

→ No electron can exist inside the nucleus

# ELECTROSTATICS

## Magnetic field (B)

The force experienced by the magnet in its surroundings is known as magnetic field, it is represented as 'B'.

Applied current of Magnetic field.

"current always conduct in closed loop"



## Magnetic flux (Φ)

magnetic field per unit area is magnetic flux  $\Phi = \frac{B}{\mu}$

## Divergence (E), (∇)

\* when density increases permittivity ↓.

$$E = \frac{\rho}{\epsilon_0} \quad \text{Coulomb Dynamics (E)}$$

$\rho \rightarrow$  density     $\epsilon_0 \rightarrow$  permittivity in vacuum.

## Magnetic flux Density.

It is the force acting per unit current, per unit length in a wire.

## Magnetic flux formula.

### (\*) magnetic flux (surface area)

It is defined as magnetic field per unit area  $\Phi_B = B \cdot A$

$$\Phi_B = B \cdot A \cos \theta$$

consider a small surface of area  $dA$  in an surface the flux through the surface is  $\Phi_B = B \cdot dA$

∴ Total flux in an surface area is the sum of individual mag flux  $\Phi_B$

$$\text{i.e., } \Phi_B = B_1 \cdot dA_1 + B_2 \cdot dA_2 \dots$$

$$\int \Phi_B = \int B \cdot dA$$

$$d_B = B \cdot A$$

$$d_B = B \cdot A \cos \theta$$

## Gauss Law in differential form.

$$\nabla \cdot B = 0$$

$\nabla \rightarrow$  Divergence

curls divergence.

what is curls divergence?

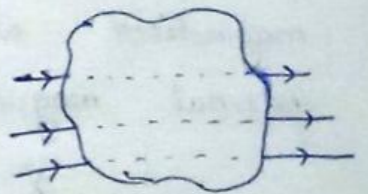
It is a theorem set which is related to the flux of a material in vector field through a closed surface area of the field in volume, and closed, enclosed.

### (\*) Gauss's Law

This law states that the amount of magnetic field lines passing through an closed surface area is zero. Because no of magnetic field lines entering inside the <sup>Gaussian</sup> surface is equal to the no of magnetic field lines goes outside.

$$\oint B \cdot ds = 0$$

$$\Phi_B = 0$$

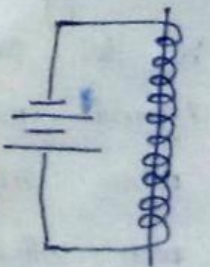


## Ampere's Circuital Law.

The law states that the no of magnetic field lines in an longitudinal section is equal to the amount of current applied.

$$\oint B \cdot dl \propto I$$

$$\oint B \cdot dl = \mu_0 I$$



8.2 - 161

8.20 - 161

8.  $\frac{dM}{dt}$



8. Faraday's law of magnetism  
It states that the circulation of magnetic field is equal to the rate of change of magnetic flux through the surface of any loop.

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \frac{d\Phi_B}{dt}$$

the no. of turns

My magnetic permeability

the relation between the magnetic field and the magnetic flux density is a material property called magnetic permeability.

Magnetic susceptibility  $\chi_m$

It is the measurement of how much a material can be magnetized when the external magnetic field is applied.

$$\chi_m = \frac{M}{H}$$

properties of magnetism

1. Diamagnetism: It is the property of a material which causes it to create a magnetic field in opposition to the external magnetic field. The material atoms possess a dipole when magnetic field is applied which is induced with such a direction and strength as to oppose the external magnetic field.

2. Paramagnetism: It is a property of a material which causes it to create the same direction external magnetic field as its own permanent dipole.

3. Ferromagnetism: It is a material property which causes it to align the atoms of magnetic field and to have permanent dipoles in the same direction.



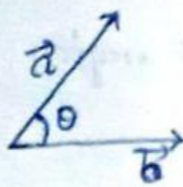
## Vector Calculus

(\*) Basic principles of vector calculus

1. dot product - / scalar product

The dot product of two vectors is defined as the product of magnitudes and cosine angle b/w them.

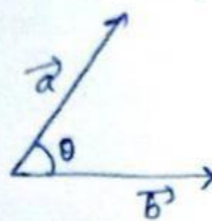
$$\vec{a} \cdot \vec{b} = ab \cos \theta$$



2. Cross product / Vector product

The cross product of two vectors is defined as the product of magnitudes and sine angle b/w them.

$$\vec{a} \times \vec{b} = ab \sin \theta$$

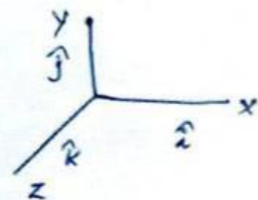


Special Cases

If there are 3-vectors  $(\vec{A}, \vec{B}, \vec{C})$  where  $\vec{C}$  is the resultant product of the vectors vector product of  $\vec{A}$  and  $\vec{B}$ .

$$\vec{C} = (\vec{A} \times \vec{B}) \times \vec{C}$$

$$\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ a_x & a_y & a_z \\ b_x & b_y & b_z \end{vmatrix}$$



$\nabla$  operators.

$\nabla f \rightarrow$  gradient  
 $\nabla \cdot \vec{a} \rightarrow$  divergence  
 $\nabla \times \vec{a} \rightarrow$  curl

When the operator acts on a scalar quantity it instructs to differentiate the scalar quantity the operator of  $\nabla$  on a scalar quantity results in vector quantity.

$$\nabla = \frac{d}{dx} \hat{i} + \frac{d}{dy} \hat{j} + \frac{d}{dz} \hat{k}$$

Gradient :- A vector quantity applied on scalar quantity is,

$$\nabla = \frac{d}{dx} \hat{i} + \frac{d}{dy} \hat{j} + \frac{d}{dz} \hat{k}$$

If  $f$  is a scalar quantity  $\nabla f = \frac{df}{dx} \hat{i} + \frac{df}{dy} \hat{j} + \frac{df}{dz} \hat{k}$ .

Qu. Find the gradient function of  $F$  at point 1, 2, 3

$$F = xy^2 + z^3y$$

$$\text{Ans } \nabla F = \frac{d(xy^2 + z^3y)}{dx} \hat{i} + \frac{d(xy^2 + z^3y)}{dy} \hat{j} + \frac{d(xy^2 + z^3y)}{dz} \hat{k}$$

$$= y^2 \hat{i} + 2xy \hat{j} + 3z^3 \hat{k}$$

$$= y^2 \hat{i} + 2xy \hat{j} + 3z^3 \hat{k}$$

$$= y^2 \hat{i} + 2xy \hat{j} + 3z^3 \hat{k}$$

$$\nabla F = y^2 \hat{i} + 2xy \hat{j} + 3z^3 \hat{k}$$

$$\text{at point } (1, 2, 3) = 2^2 \hat{i} + 2 \times 1 \times 2 \hat{j} + 3 \times 3^3 \hat{k}$$

$$= 4 \hat{i} + 4 \hat{j} + 81 \hat{k}$$

$$= 4 \hat{i} + 4 \hat{j} + 81 \hat{k}$$

Basics of Divergence.

It is a scalar quantity

It is applied to the vector quantity

$$\nabla \cdot \vec{F} = \frac{dF_x}{dx} + \frac{dF_y}{dy} + \frac{dF_z}{dz}$$

Formula: This rule states that volume integral = Surface integral.

$$\vec{\nabla} \cdot \vec{F} \Delta V = \oint \vec{F} \cdot d\vec{s}$$

$$\vec{\nabla} \cdot \vec{F} \Delta V = \oint \vec{F} \cdot d\vec{s}$$

Ques Find the divergence of the function at the point (2,1)

$$\vec{F} = xy^2\hat{i} + y^3\hat{j} + xz\hat{k} \text{ (vector function)}$$

find  $\nabla \cdot \vec{F}$

$$\Delta F = \frac{dF}{dx}\hat{i} + \frac{dF}{dy}\hat{j} + \frac{dF}{dz}\hat{k}$$

$$= y^2\hat{i} + 3y^2\hat{j} + x\hat{k}$$

$$\text{at } (2,1) = 4\hat{i} + 3\hat{j} + \hat{k} = 4 + 3 + 1 = 8$$

Curl function

it is a vector quantity where it is applied to another vector quantity

$$\text{Curl of } \vec{F} = \nabla \times \vec{F}$$

$$\vec{F} = F_1\hat{i} + F_2\hat{j} + F_3\hat{k}$$

$$\nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ F_1 & F_2 & F_3 \end{vmatrix}$$

$$\nabla \times \vec{F} = \lim_{\Delta S \rightarrow 0} \frac{\oint \vec{F} \cdot d\vec{l}}{\Delta S}$$

$$\Delta S \nabla \times \vec{F} = \lim_{\Delta S \rightarrow 0} \oint \vec{F} \cdot d\vec{l}$$

Surface area  
Integral

Lineal  
Integral

This is known as Stokes theorem

Ques Volume Integrals

It is a representation of vector point function and volume (V) enclosed by a closed surface. S

Ques Find the volume integral of  $F = 2x^2 - x\hat{j} + y\hat{k}$ , given boundary surface

$$(x=0, x=2, z=0, z=2)$$

$$y=0, y=4$$

$$z=0, z=2$$

Ans. Solution  $\iiint_V r \cdot dV \quad dx, dy, dz$

$$\int_0^2 \int_0^4 \int_0^2 [2x^2 - x\hat{j} + y\hat{k}] dx dy dz$$

$$\int_0^2 \int_0^4 \left[ \frac{2x^3}{3} - x\hat{j} + y\hat{k} \right]_0^2 dy dz$$

$$= \int_0^2 \left[ \left( \frac{2x^3}{3} - x\hat{j} + y\hat{k} \right) \right]_0^4 dx$$

$$= \int_0^2 \left[ \left( \frac{2x^3}{3} - x\hat{j} + 4\hat{k} \right) - \left( \frac{2x^3}{3} - x\hat{j} \right) \right] dx$$

$$= \int_0^2 \left[ \left( \frac{2x^3}{3} + 4\hat{k} \right) - \left( \frac{2x^3}{3} - x\hat{j} \right) \right] dx$$

$$= \int_0^2 \left[ \frac{2x^3}{3} + 4\hat{k} + x\hat{j} - \frac{2x^3}{3} \right] dx$$

$$= \int_0^2 \left[ x\hat{j} + 4\hat{k} \right] dx = \left[ \frac{x^2}{2}\hat{j} + 4x\hat{k} \right]_0^2$$

$$= \left[ \frac{4}{2}\hat{j} + 8\hat{k} \right] - [0] = 2\hat{j} + 8\hat{k}$$

# SEMICONDUCTOR

8 part SA  
define, peculiarity, Appli

## Super Conductivity & Conductors:

materials having zero resistance = Super conductor.

The phenomena exactly zero resistance in a material is known as super conductive material.

(\*) Critical temperature: for a normal conductor, resistance is function of temperature therefore  $R = f(T)$  (As temp increase resistance also increase)  
The temp at which resistance turns to zero <sup>(infinite conductivity)</sup> is called critical temp/<sub>trans-</sub>istant temperature.  
Case-1

When temp decreases the Resistance of material is lower down (non-zero) and infinite conductivity. Such materials are known as Super conductors.

(\*) Above the critical temp the material will be in normal state. Super conductivity is in reversible process so <sup>that</sup> when temp is increased from the critical temp hence the resistivity also increases. Thus it is known as reversible process.

## Meissner Effect

The phenomena of expulsion of magnetic field lines from superconductors is known as Meissner's effect.

$$B = \mu_0 (H + M)$$

Taking H outside

$$B = \frac{\mu_0 H}{\mu_0} \left( 1 + \frac{M}{H} \right) \quad \text{--- (2)}$$

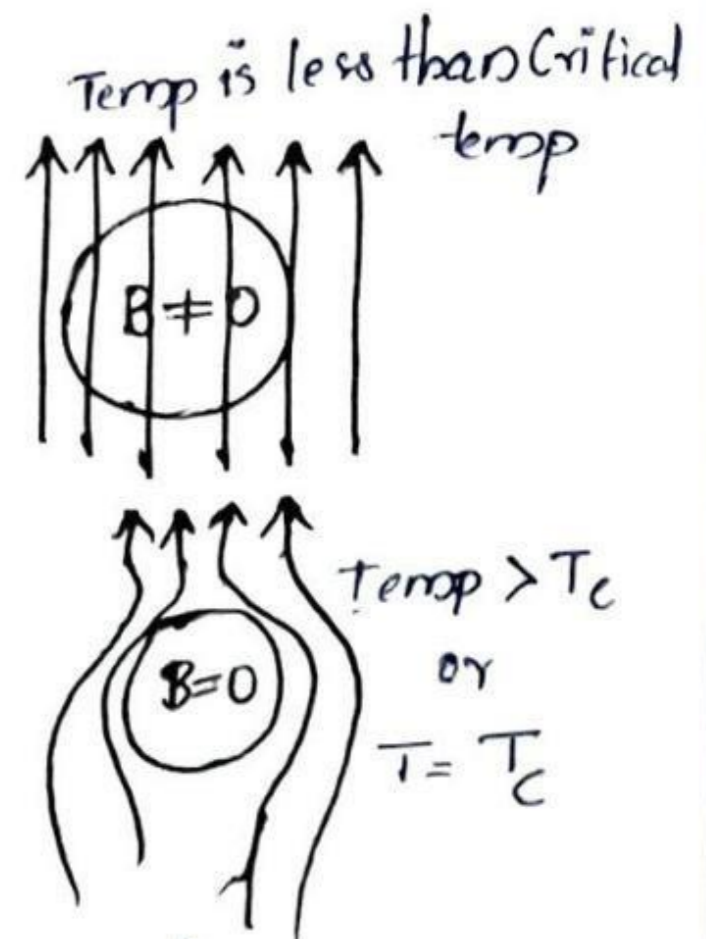
We know that  $\frac{M}{H} = \chi$  apply in eqn (2)

$$0 = \mu_0 H \left( 1 + \frac{M}{H} \right)$$

$$1 + \chi = 0$$

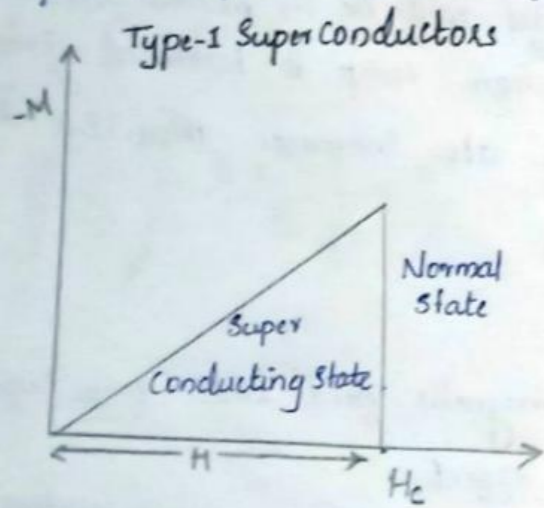
$$\chi = -1$$

for diamagnetic material.

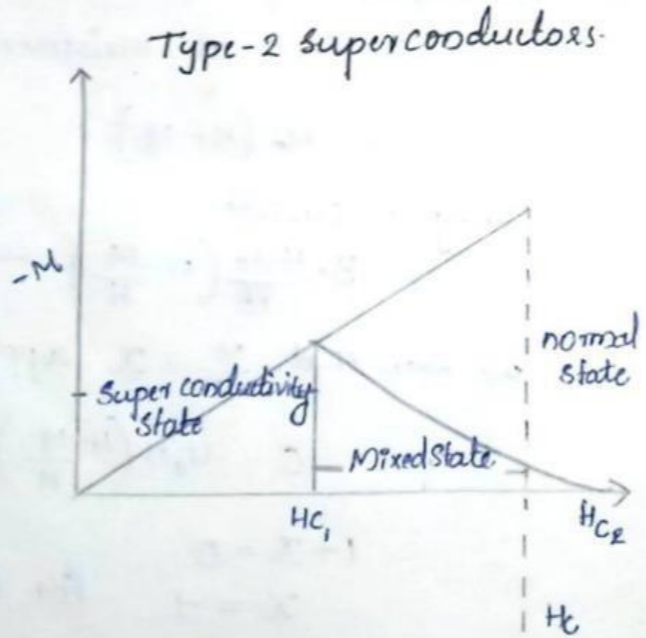


Type-1 Superconductors	TYPE-2 Superconductors.
(*) The material loses its magnetisation after	(*) it loses its magnetisation gradually.
(*) It exhibits complete Meissner effect	(*) It doesn't exhibit Meissner effect.
(*) It exhibits only one critical mag - field.	(*) It <del>is not mixed</del> exhibits diff critical magnetic field.
(*) It is not mixed state	(*) It is mixed state.
(*) They are called soft superconductors	(*) They are hard superconductors.
(*) eg:- Aluminium (Al), Indium (In), Tin (Sn), Lead (Pb)	(*) eg:- Germanium (Ge), Niobium (Nb), Vanadium (Vn)

Graphical representation Type 1 and Type 2 Superconductors



$H_c \rightarrow$  Critical mag  
 $M \rightarrow$  magnetising field



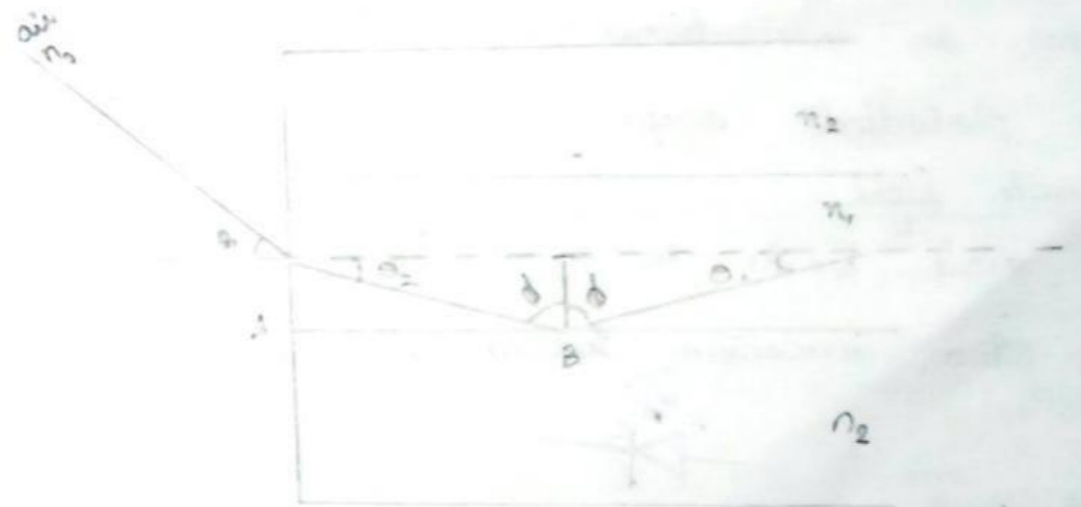
## Numerical Aperture

- (\*) In optics numerical aperture is defined as non-dimensional number that characterises the range of angles over which the system can accept or emit light.
- (\*) It is the relation b/w acceptance angle and refractive index of 3 media involved, core, cladding, air. The light ray is incident on the fibre core. cent. (centre of the fibre) at an angle  $\theta_1$ , which is less than the acceptance angle of the fibre. The ray enters from the air medium (refractive index  $n_0$ ) and the fibre for refractive index ( $n_1$ ) which is slightly greater than cladding refractive index ( $n_2$ ). The ray is normal to the axis, by consider the refraction of air-core by using (Snell's law)

$$\frac{\sin i}{\sin r} = \frac{n_2}{n_1}$$

$$n_1 \sin i = n_2 \sin r$$

$$n_0 \sin \theta_1 = n_1 \sin \theta_2$$



## LED

A p-n junction diode which operates as forward biased the  $e^-$  from the n-region and goes from p-region meet at the junction formed as depletion region this  $e^-$  recombine with holes in this region. Just formation of  $e^-$  from the conduction band to the empty state of valence band and it releases photon.

$$E_g = \frac{hc}{\lambda}$$

**PHOTO DIODE**: The diode which produce current when light falls on it.

- (1) It is a light sensitive device which converts light energy  $\rightarrow$  electrical energy.
- (2) It is a p-n junction diode where n and p and it is made an interstitial layer called depletion layer.
- (3) The photodiode accept the light as an input source to generate light.
- (4) The diode works in reverse biased condition.
- (5) eg: Silicon, Germanium, Selenium, Gallium.



Applications: Camera, scanner.

## BCS Theorem (Cooper pair)

It was developed by Bardeen and Cooper and Schrieffer and this theorem is based on super pair  $e^-$ .



Consider an  $e^-$  moving in a lattice. The other ions are attracted towards the  $e^-$  due to Coulomb force as the positive charge is positive and this type of interaction is known as  $e^-$ -phonon interaction. Now the region of increased charge density attracts another  $e^-$  and eventually around  $e^-$  forms a cloud that is known as  $e^-$  lattice. The interaction between  $e^-$  and  $e^-$  through the phonon is attractive. The  $e^-$  cloud is known as Cooper pair.

### Condition

The temperature decreases below  $T_c$  (superconducting transition temperature) and high temp can destroy super pair.

**HCBM** High Temperature Super Conductor Material (HTSC)

### Applications of Super Conductivity

In medical, laser technology, metrology.

### Fiber Optics

Optical fiber cable



• It is a core made of total internal reflection.

$$\sin \theta_c = \frac{n_2}{n_1}$$

• The core has angle of incidence  $\theta_i$  greater than critical angle.

∴ critical angle formula  $\frac{\sin \theta_c}{\sin 90^\circ} = \frac{n_2}{n_1}$

$$\sin \theta_c = \frac{n_2}{n_1}$$

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right)$$

### Step Index Fibre and Graded Index Fibre

Step Index Fibre	Graded Index Fibre
It has const refractive index	Refractive index reduces gradually
refractive index reduces from $n_1$ to $n_2$ suddenly	And for cladding core boundary the difference of refractive index is small.
Signals can be trans	
long distance transmission of light	Short distance transmission of rays.
zig-zag path is followed by light ray.	Spherical or helical path is followed by light ray.

### Accental Centre and Numerical apperture.

Stability and gathering of light is termed numerical apperture.

$$NA = \sin^{-1} \sqrt{n_1^2 - n_2^2}$$

$$\frac{\sin i}{\sin(90^\circ - \theta)} = \frac{n_2}{n_1}$$

$$\frac{\sin i}{\cos \theta} = \frac{n_2}{n_1}$$

Let  $n = \mu$  ...  $\frac{\sin i}{\cos \theta} = \frac{\mu_2}{\mu_1}$

$$\frac{\sin \theta}{\cos \theta} = \frac{\mu_2}{\mu_1} \quad \text{Where } \mu_2 \rightarrow \text{angle of refractive index of 2nd medium}$$

$$\tan \theta = \frac{\mu_2}{\mu_1} \quad \mu_1 \rightarrow \text{refractive index of 1st medium}$$

if  $\mu_1 = 1$   $\frac{\sin \theta}{\cos \theta} = \mu_2$   $\begin{matrix} i \rightarrow \\ 0 \rightarrow \end{matrix}$

$$\sin \theta = \mu_2 \cos \theta \rightarrow \mu_2 = \mu_1 \cos \theta$$

if  $\mu_2 = \mu_1$ , replacing  $\mu_2$  as  $\mu_1$  the above equation

$$\sin \theta = \mu_1 \cos \theta \quad (2)$$

by apply snells law

$$\frac{\sin \theta}{\sin 90^\circ} = \frac{\mu_2}{\mu_1}$$

$$\sin \theta = \frac{\mu_2}{\mu_1} \quad \theta = \sin^{-1}\left(\frac{\mu_2}{\mu_1}\right)$$

$$\mu_2 = \mu_1 \sin \theta$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\cos \theta = \sqrt{1 - \sin^2 \theta}$$

$$\cos \theta = \sqrt{1 - \left(\frac{\mu_2}{\mu_1}\right)^2}$$

$$\sin \theta = \mu_1 \cos \theta$$

$$\sin \theta = \mu_1 \sqrt{1 - \frac{\mu_2^2}{\mu_1^2}}$$

$$\sin \theta = \sqrt{\mu_1^2 - \mu_2^2}$$

$$\theta = \sin^{-1} \sqrt{\mu_1^2 - \mu_2^2}$$

$$\text{Or } NA = \sin^{-1} \sqrt{\mu_1^2 - \mu_2^2}$$

$$\oint_V \rho dv = \epsilon_0 \oint_S \vec{E} \cdot d\vec{s}$$

$$D = \epsilon_0 \vec{E}$$

$$\oint_S D \cdot d\vec{s} = \int_V \rho dv$$

electric flux divergence in Maxwell equation

electric displacement vector in free space by using Gauss divergent theorem.

$$\oint_S D \cdot d\vec{s} = \int_V (\nabla \cdot \vec{D}) dv$$

$$\int_V \rho dv = \int_V (\nabla \cdot \vec{D}) dv$$

$$\rho dv = \nabla \cdot \vec{D} dv$$

$$\int_V (\nabla \cdot \vec{D} - \rho) dv = 0$$

$$\text{div } \vec{D} = \rho$$

$$\nabla \cdot \vec{D} = \rho$$

$$d\vec{E} = \frac{\rho}{\epsilon_0}$$

$$d\vec{E} = \vec{E} \cdot d\vec{s}$$

$$\rho = \rho / dv$$

$$\frac{\rho}{\epsilon_0} \int_V dv = \vec{E} \cdot d\vec{s}$$

$$\rho \int_V dv = \epsilon_0 \vec{E} \cdot d\vec{s}$$

$$D = \epsilon_0 \vec{E}$$

$$\rho \int_V dv = \epsilon_0 \vec{E} \cdot d\vec{s}$$

$$\rho \int_V dv = \epsilon_0 \int_V \nabla \cdot \vec{E} dv$$

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Assignment

Time independent EQUATION

MAXWELL'S CURL

According to Faraday's law of magnetism  $e = -\frac{d\phi}{dt}$  — (1)

$$e = \int \vec{E} \cdot d\vec{l} \quad \text{--- (2)}$$

$$\phi = \int \vec{B} \cdot d\vec{s} \quad \text{--- (2)}$$

Sub eqn (2) in (1)

$$-\frac{d\phi}{dt} = \int \vec{E} \cdot d\vec{l}$$

$$e = -\frac{d}{dt} \left( \int \vec{B} \cdot d\vec{s} \right)$$

From eqn (2) and (3)

$$\int \vec{E} \cdot d\vec{l} = \int \text{curl } \vec{E} \cdot d\vec{s}$$

$$\text{So, } \int \text{curl } \vec{E} \cdot d\vec{s} = \int \frac{d}{dt} (\vec{B} \cdot d\vec{s}) = 0$$

$$\int \left[ \text{curl } \vec{E} \cdot d\vec{s} + \frac{d}{dt} (\vec{B} \cdot d\vec{s}) \right] = 0$$

Take 'ds' commonly outside

$$\int \left[ \text{curl } \vec{E} + \frac{d\vec{B}}{dt} \right] \cdot d\vec{s} = 0$$

$$\text{curl } \vec{E} = -\frac{d\vec{B}}{dt}$$

$$\text{or } \nabla \times \vec{E} = -\frac{d\vec{B}}{dt}$$

→ s is the arbitrary, equation is valid when integration is zero

2) By Gauss Law in Magnetism the net mag flux emerging out of a enclosed surface area is equal to zero

$$\text{Integral of } \int \vec{B} \cdot d\vec{s} = 0$$

$$\int_S \vec{B} \cdot d\vec{s} = \int_V \nabla \cdot \vec{B} dv$$

$$\int_S \vec{B} \cdot d\vec{s} = \nabla \cdot \vec{B}$$

$$\text{So } \nabla \cdot \vec{B} = 0$$

According to Ampere's circuital law

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I$$

where  $I = \int \vec{j} \cdot d\vec{l}$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \int \vec{j} \cdot d\vec{l}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 I \quad \text{--- (1)}$$

$$\text{Current through density } \vec{j} = \int \vec{j} \cdot d\vec{l} \quad \text{--- (2)}$$

$$\oint \vec{B} \cdot d\vec{l} = \mu_0 \int \vec{j} \cdot d\vec{l}$$

By using Stokes theorem

$$\oint \vec{B} \cdot d\vec{l} = \int \vec{\nabla} \times \vec{B} \cdot d\vec{S}$$

$$\int \vec{\nabla} \times \vec{B} \cdot d\vec{S} = \mu_0 \int \vec{j} \cdot d\vec{S}$$

$$\text{curl } \vec{B} = \mu_0 \vec{j} \quad \text{--- (3)}$$

By equation of continuity

$$\text{div } \vec{j} + \frac{d\rho}{dt} = 0$$

$$\text{div } \vec{j} = -\frac{d\rho}{dt}$$

$$\text{curl } (\text{curl } \vec{B}) = \text{curl } \vec{j} \quad \text{--- (4)}$$

$$\text{curl } (\text{curl } \vec{B}) = \text{curl } \vec{j}$$

$$\text{curl } (\text{curl } \vec{B}) = \text{curl } \vec{j}$$

while introducing a new term  $\vec{D}$  in the modified equation 4

$$\text{curl } \vec{H} = \vec{j} + \vec{j}_d \quad \text{--- (5)}$$

$\vec{j}_d$  is a introduced by Maxwell and known as displacement

current density

$$\text{curl } \vec{H} = \vec{j} + \vec{j}_d$$

$$\text{curl } (\text{curl } \vec{H}) = \text{curl } (\vec{j} + \vec{j}_d)$$

$$\text{curl } (\text{curl } \vec{H}) = \text{curl } \vec{j} + \text{curl } \vec{j}_d$$

from Maxwell's 2 equation

$$\text{curl } \vec{H} = \vec{j} + \vec{j}_d$$

By 1st eqn of Maxwell

$$\text{curl } \vec{H} = \vec{j} + \vec{j}_d$$

$$\text{curl } (\text{curl } \vec{H}) = \text{curl } (\vec{j} + \vec{j}_d) \quad \text{--- (6)}$$

$$\text{curl } (\text{curl } \vec{H}) = \text{curl } \vec{j} + \text{curl } \vec{j}_d$$

By 2nd eqn of Maxwell

$$\text{curl } \vec{H} = \vec{j} + \vec{j}_d$$

$$\text{curl } (\text{curl } \vec{H}) = \text{curl } (\vec{j} + \vec{j}_d)$$

Conduction current: movement of charges in a conductor when a field is applied. By Ohm's law  $\vec{j} = \sigma \vec{E}$  and  $\vec{E} = -\vec{\nabla} \phi$

The continuity of conductor is defined  $\vec{j} = \frac{dQ}{dt}$

$$\vec{j} = \frac{dQ}{dt}$$

$$\vec{j} = \sigma \vec{E}$$

$$\vec{j} = \sigma \vec{E}$$

### Displacement Current

A time varying  $\vec{E}$  field is considered as an equivalent of  $\vec{j}$  which with change of  $\vec{E}$  and hence  $\vec{j}_d = \epsilon_0 \frac{d\vec{E}}{dt}$

$$\vec{j}_d = \epsilon_0 \frac{d\vec{E}}{dt}$$

Displacement current



2

displacement current  $I_d = A \frac{dQ}{dt}$   $D \rightarrow \epsilon_0 E$   
 displacement current density,  $J_d = \frac{dD}{dt}$

connection of displacement current with conduction current  
 displacement current is the current i.e., set up in a dielectric medium due to variation of induced displacement of charge.

$I_c = \frac{V}{R}$   $Q = CV$

$I = \frac{Q}{t}$

$\therefore I_d = \frac{dQ}{dt} = \frac{d(CV)}{dt}$

$I_d = C \frac{dV}{dt}$

$j = I/A$  current charge density =  $j$

$\therefore J_c = I_c/A = \sigma E$

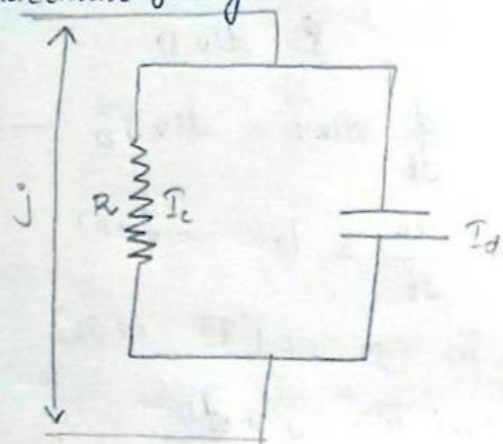
$E = \frac{V}{d}$   $C = \frac{\epsilon_0 A}{d}$   $D = \epsilon_0 E$

$J_d = \frac{I_d}{A} = \frac{C}{A} \frac{dV}{dt} = \frac{\epsilon_0 A}{dA} \frac{dV}{dt} = \frac{\epsilon_0 V}{d} = \frac{\epsilon_0 E d}{dt}$

$J_D = \epsilon_0 E$

$J_d = \frac{dD}{dt}$   $J_c = \frac{I_c}{A} = \sigma E$

Total =  $J_c + J_d$   
 (Current charge density)



### Velocity of EM-wave In Free Space.

Assume according to Maxwell's assumption the velocity of EM waves in free space is,

$V = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$  — (1)

proof :- Maxwell's equation assumes the simpler form.

$\text{div } \vec{D} = 0$  — (2)

There is no free charge

$\text{div } \vec{B} = 0$  — (3)

$\text{curl } \vec{E} = - \text{curl } \frac{d\vec{B}}{dt}$  — (4)

$\text{curl } \vec{H} = \frac{d\vec{B}}{dt}$  — (5)

$J = 0$  there is no conduction current

multiply eqn (4) by curl

$\text{curl}(\text{curl } \vec{E}) = \text{curl} \frac{d\vec{B}}{dt}$  — (6)  $(B = \mu H)$

$\text{curl}(\text{curl } \vec{E}) = \text{curl} \frac{d\mu \vec{H}}{dt}$

$= - \text{curl} \frac{d\mu \vec{H}}{dt}$

$= -\mu \frac{d}{dt} \text{curl } \vec{H}$

$= -\mu \frac{d\vec{B}}{dt} \times \frac{dD}{dt}$

$= -\mu \frac{d}{dt} \times \frac{d\epsilon_0 E}{dt}$

$= -\mu \epsilon_0 \frac{d}{dt} \left( \frac{dE}{dt} \right)$

double differentiation

$= -\mu \frac{d^2}{dt^2} (E)$

$$= -\mu\epsilon \frac{d^2 \mathbf{E}}{dt^2} \quad \text{--- (6)}$$

$$\text{curl}(\text{curl} \mathbf{E}) = -\mu\epsilon \frac{d^2 \mathbf{E}}{dt^2} \quad \text{grad} \rightarrow \text{gradient}$$

$$\text{curl}(\text{curl} \mathbf{E}) = \text{gradient}(\text{div} \mathbf{E}) - \nabla^2 \mathbf{E}$$

$$\text{div} \mathbf{D} = 0 \quad \therefore \text{div} \mathbf{E} = 0$$

$$\text{curl} \text{curl} \mathbf{E} = -\nabla^2 \mathbf{E} \quad \text{--- (7)}$$

$$-\mu\epsilon \frac{d^2 \mathbf{E}}{dt^2} = -\nabla^2 \mathbf{E} \quad \text{--- (8)}$$

$$\mu\epsilon \frac{d^2 \mathbf{E}}{dt^2} = \nabla^2 \mathbf{E}$$

Eqn no (8) is well known as differential eqn

This shows that  $\mathbf{E}$  is propagated as a wave with  
therefore the above eqn is simplified in the form of

$$v = \frac{1}{\sqrt{\mu\epsilon}}$$

$$\nabla^2 \mathbf{H} = \mu_0 \epsilon_0 \frac{d^2 \mathbf{H}}{dt^2}$$

$v = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$  This is the velocity of EM waves. Velocity is

$$v = \left( \frac{1}{(4\pi \times 10^{-7} \times 8.85 \times 10^{-12})} \right)^{1/2} = 2.997 \times 10^8 \text{ m/s}$$

Poynting's theorem

The rate of energy transfer (or product) from a region of space equals  
to the rate of work done on a charge distribution + energy  
flux leaving that region.

$$-\frac{d\mathbf{u}}{dt} = \nabla \cdot \mathbf{E} + \mathbf{j} \cdot \mathbf{E}$$

Maxwell's Equations (4 equations)

Gradient, divergence, curl, Gauss divergence theorem, Stokes' theorem,  
Theorem and its proofs

Test :- Maxwell's Equations (4 equations)